# CONTENTS

**NORTHWESTERN UNDERGRADUATE RESEARCH JOURNAL**

**FEATURES**

**PROFILE**  
In Search of Biomaterials 5  
Preventing the Next Plague 7  
The Logic Behind Lyrica 9  

**REPORT**  
An Error Analysis of the Phased Array Antenna Pointing Algorithm for STARS Flight Demonstration #2  
Michael P. Carney

**RESEARCH**

Characterizing Cell Motility of Cancerous Neuronal and Progenitor Cells 19  
Kwan Y. Chen

The Physics and Applications of Index Patterned Fabry-Pérot Lasers 25  
James M. Rondinelli

Transverse Vibrating Modes of a Resonance Wire Loop 33  
Aysha Chowdhry & Elaine Tsao

Decoupled Control for the Snakeboard 38  
Benjamin Jay Stephens

Investigations of Nanoscale Ferromagnetism in In/As/InMnAs Core/Shell Nanowires Using Magnetic Force Microscopy 46  
Dinna G. Ramlan

Removing Quantization Artifacts in Color Images Using Bound Interval Regularization 55  
Tom Yu Duyang

Evaluating Sam: Iterative Design of a Story Listening System and Embodied Conversational Agent 61  
Candice W. Yee

---

About NURJ 2  
Acknowledgments 2  
From the Editor 3  
About the Contributors 67
ACKNOWLEDGMENTS

The Northwestern Undergraduate Research Journal could not have been possible without the assistance of many people. We would like to thank them all for their contributions, patronage, and resources. First, we owe huge thanks to Henry S. Bienen, President of Northwestern University, for his support and suggestions. Thanks also to the Office of the President, along with Steve Fisher and the Office of the Provost, Weinberg College of Arts and Sciences, and McCormick School of Engineering and Applied Science, for their generous financial contributions, which are a testament to the importance and value of research at this university. We would also like to acknowledge the members of the Faculty Review Board: Allen Taflove, Richard Gaber, Ming-Yang Kao, Fabian Bustamante, Mark Hersam, Mary Phillips, Paul Umbanhowar, Michael Peshkin, and Mike Roloff. Their time and expertise came at an important step in the process. Special thanks as well to Allen Taflove for serving as our advisor. Next, we would like to recognize all of the contributors for sharing their exemplary research, along with their advisors and mentors for their inspiration and guidance. We also express thanks to Patrick Stansbury at Pentagon Publishing for helping us despite late notice, and to Kate Igoe and the Office of Corporate Relations for facilitating the distribution of journals. Finally, we thank the entire NURJ staff for their hard work, dedication, and talents.

nurj
Northwestern Undergraduate Research Journal
Volume 2, May 2005

EDITOR Laura Hughes
SUBMISSIONS Thomas O'Sullivan
STUDENT & FACULTY RELATIONS Paul Karagiannis
FINANCES Paul Balash
DESIGN & LAYOUT Vinhfield Ta
cO-DESIGN Alexander Hertel-Fernandez
COPY EDITING Caitlin Brown
Lila Schwartz
TECHNOLOGY Jay Zeschin
FACULTY ADVISOR Allen Taflove

STUDENT REVIEW BOARD
Paul Balash, Alexander Hertel-Fernandez, Laura Hughes, Jessica Hurst, Paul Karagiannis, Thomas O'Sullivan, Jerome Pandell, Adria Pathhoff, James Rondinelli, Lila Schwartz, Vinhfield Ta, Jay Zeschin

FACULTY REVIEW BOARD
Fabian Bustamante Computer Science
Richard Gaber Biochemistry, Molecular & Cell Biology
Mark Hersam Materials Science & Engineering
Ming-Yang Kao Computer Science
Michael Peshkin Mechanical Engineering
Mary Phillips Electrical & Computer Engineering
Mike Roloff Communication Studies
Allen Taflove Electrical & Computer Engineering
Paul Umbanhowar Physics & Astronomy

MISSION STATEMENT
The Northwestern Undergraduate Research Journal provides students the opportunity to experience writing and submitting original research. The goals of the journal are to encourage the dissemination of ideas, increase undergraduate involvement in research, and recognize the impressive work being performed by undergraduates.

Publisher Triangle Printers, 3737 Chase Avenue, Skokie, Illinois 60076
www.triangleprinters.com
Advertising Pentagon Publishing, P.O. Box 451403, Atlanta, Georgia 31145
www.pentagon-usa.com
Cover Art by Laura Hughes, Vinhfield Ta, and Alexander Hertel-Fernandez
Photography Pictures by Paul Balash

NURJ Office
Technological Institute M471
2145 Sheridan Road
Evanston, Illinois 60208

Email nurj@northwestern.edu

Online Publication and Website http://www.nurj.org

NORTHWESTERN UNIVERSITY
From the editor

To the Northwestern Research Community:

It gives me great pleasure to present to you the second annual edition of the Northwestern Undergraduate Research Journal. NURJ is part of an increasing effort to involve and recognize undergraduates in cutting-edge research. Here at Northwestern, the willingness of professors to teach undergraduates in their laboratories has created unprecedented opportunities for students. In the scientific and engineering disciplines, research is a critical part of a Northwestern education. This experience allows students to apply the theoretical knowledge learned in the classroom to practical problems.

Moreover, the journal seeks to improve communication within disciplines. Problems we are now facing in the scientific community, such as the treatment of cancer, require complex solutions based on interdisciplinary work. As a result, the ability to express the purpose, results, and conclusions of research becomes an important skill for students.

The submissions to the journal were evaluated based on the clarity, purpose, and scientific merit of the research. After a student review board had expressed their opinions on these criteria, a faculty review board assessed the research. Students were then allowed to improve their papers based on our cumulative suggestions. The submissions included in this year's journal reflect the high-quality research being performed by Northwestern undergraduates.

This year, in addition to research articles from undergraduates, we have also included three profiles on professors who have inspired our staff: Professors Guillermo Ameen, Robert Lamb, and Richard Silverman. I hope that you enjoy reading the articles.

Sincerely,

Laura Hughes
Editor
DO YOU HAVE

Submit to the
Northwestern Undergraduate Research Journal
Now accepting submissions for the 2006 publication

IN YOU?

Information and instructions are available at http://www.nurj.org
Want to become a part of NURJ? Learn more by contacting us at nurj@northwestern.edu

SCHREIBER
Would you like to own your future? You can at Schreiber Foods. As the world’s largest private-label cheese company, we’re continuously seeking bright, talented individuals to be contributing members of our team. Every year we’re committed to hiring the best. We recruit a variety of disciplines for full-time and internship positions in all areas of our business. We offer excellent opportunities that provide individuals with challenges and professional growth.

Schreiber Foods Inc.
PO Box 19010
Green Bay, WI 54307-9010
920-437-7601 (phone)
800-344-0333 (toll-free)
920-437-1617 (fax)
www.schreiberfoods.com
in SEARCH of BIOMATERIALS

by Laura Hughes

In fields such as medicine, developing technologies often require interdisciplinary work to be effective. Dr. Guillermo Ameer offers one example of interdisciplinary research being performed at Northwestern University. His research fuses biomaterials, tissue engineering, and biotechnology to address medically relevant questions. Ameer, a professor in Biomedical Engineering, also has affiliations with the Interdepartmental Biological Sciences (IBiS), the Institute of Bioengineering and Advanced Medicine in the Feinberg School of Medicine, and the Department of Medicine Evanston-Northwestern Healthcare at Evanston Hospital.

Ameer's lab focuses on three main projects. First, they are developing several biomaterials to improve tissue engineering. These new biomaterials could be used to create small blood vessels, heart valves, or ligaments in the knee. In practice though, the issue becomes more complex, since the body can reject tissues if it deems them foreign. As a result, biocompatibility becomes a major issue in developing artificial tissues. Furthermore, degradation of the biomaterial should not produce toxic byproducts, or else unwanted side effects develop.

Secondly, Ameer, in collaboration with Dr. Vadim Backman, another Biomedical Engineering professor, is using new biomaterials to improve molecular imaging. Together, they are using gold nanoshells to non-invasively gather information about the biology and pathology of cells. This method is based on a technique called four-dimensional elastic light scattering fingerprinting (4D-ELF), developed by Backman. Light is scattered within a tissue based on the amount of absorption of light and the amount of scattering within the tissue. Absorption is influenced by molecules within the blood, like hemoglobin, while scattering is determined by organelles or macromolecular complexes. Since the light scattering changes dramatically when the tissue becomes altered, like in disease states, this method can detect structures 10-20 times smaller than what is used in conventional methods. Ameer is currently using this method to study how his engineered tissues mimic natural ones.

Lastly, Ameer is developing a method to improve blood purification. When a patient loses his or her kidney function, he or she often must undergo dialysis, a non-specific process. As a result, the group is trying to develop a way to improve hemodialysis by filtering the blood without losing compounds or proteins that are beneficial to the patient. With conventional technologies to imitate kidney function, these molecules are removed from the blood by simply passing the blood through a strainer, which prevents big molecules from being removed. However, this method has two major problems: desirable small molecules are removed, and unwanted large molecules are not, since they cannot pass through the sieve. Ameer's method uses single-chain antibody fragments to remove specific molecules from the blood. Since antibodies are highly specific for one molecule, this method could then potentially remove only the molecules that need to be removed while leaving the beneficial molecules.

Since many of his projects depend on collaboration between professors with different areas of expertise, Ameer highlights Northwestern's
Left: Bone screws created by Ameer's lab biodegrade quicker than current ones, thereby improving treatment options.
Center: Porous biodegradable elastomeric scaffold for engineering small diameter blood vessels.
Right: Scanning electron micrograph of an engineered biodegradable scaffold.

MASTERS COURTESY GULSHAN AMEEDE

Flexibility as one of its strengths. Ameer, for example, collaborates with faculty at the Feinberg School of Medicine, Evanston-Northwestern Healthcare, as well as within Northwestern. "People are excited about your work, so it's easy to make connections that we need to do the work that we do, which is very interdisciplinary," he explained.

Additionally, the environment at NU balances hard work with other things, creating a less-stressed environment. "You have a lot of smart people, but it's a laid-back environment—not a high pressure cooker," Ameer said. "That can be good in a number of ways."

Northwestern's environment also allows undergraduates to become involved in research. Ameer feels that the undergraduates performing independent research benefit from the experience. "Working in a lab gives students a very good opportunity over their peers, especially when going into graduate school," he explained. "People at Northwestern are open-minded about having folks work in their lab."

Sadiya Khan, a junior working in Ameer's lab, agrees that the lab experience has been a positive one: "This experience has given me a lot of practical skills with regards to biological research," she said. "This positive research experience has led me to want to continue doing research after graduation, during medical school." Khan was initially attracted to Ameer's lab because of the research topics. "When picking my [Biomedical Engineering] concentration, tissue engineering was one of the things that interested me," Khan said. "By joining the Ameer lab I was able to more concretely explore the meaning of tissue engineering."

In contrast, Ameer himself began his career as a chemical engineer, but decided that he preferred to go into research rather than industry: "I wasn't interested in traditional Chemical Engineering. I always had an interest in helping people, and people need help most with their health."

Although Ameer's projects are currently in the developmental phase, he hopes that some will eventually translate into changes in medicine. "It's an exciting field to be in—there's lots of interesting research out there." The discovery of new research has been aided by media coverage, which has influenced governmental funding. "It's about 180 degrees from where I was when I was going into graduate school," Ameer proclaimed. However, the media has also increased expectations; Ameer feels that some of these expectations are unrealistic in the near future.

"You have to keep things in check; this is a very new field," he said. "However, it has the potential to do a lot of good things."
Preventing the Next Plague

By Alexander W. Hertel-Fernandez

As I await the interview with Dr. Robert Lamb, I glance across the walls of his lakefront Cook Hall office. There are two walls dedicated to housing the requisite assortment of past theses, texts, journals, and laboratory protocols. Adorning the two remaining walls, amongst tastefully framed pictures of family and Chicago skylines is an intimidating collection of professional accolades—Dr. Robert A. Lamb, President of the American Society for Virology; Editor-in-Chief, Virology; Fellow, American Academy of Microbiology—the collection continues to nearly every organization or journal related to the field of virology. This is clearly a man that has dedicated his life to the pursuit of knowledge and to the improvement of humankind through dissemination of that knowledge.

Professor Lamb’s research focuses on the life cycles of influenza and paramyxoviruses, two families of viruses that cause dangerous diseases and great human suffering. Lamb’s work specializes in fusion, entry and the subsequent innate immune response to viruses. This research can further be divided into four categories: structure and replication of virion particles, virus fusion and assembly, structure and function of the influenza ion channels, and viral anti-interferon response. One of the highlights of the research being undertaken in the Lamb lab is their work with the structure and function of the paramyxovirus fusion protein.

Paramyxovirus entry into cells is mediated by the viral fusion protein. In order for this process to occur, this glycoprotein must undergo cleavage that activates the fusion ability of the virus; without such cleavage, infectivity is minimal. Despite the importance of this conformational change, little was known until very recently about the structure of the fusion protein. Dr. Lamb, in collaboration with the laboratory of Dr. Theodore Jardetzky, a structural biologist at Northwestern, has successfully resolved the crystal structure of the key components of the fusion protein, namely the core trimer. Beyond the direct applications to the study of paramyxoviruses, this discovery has also provided valuable insight in the taxonomy and evolution of viruses. Although the fusion proteins of comparable viruses, such as influenza, HIV-1, and Ebola distinctly vary in size, structure, and chemical composition, all three have similarities to the core fusion protein of the paramyxovirus. This sheds new light to the theory of common evolution and subsequent mechanism of fusion and infection of human cells among viruses.

Born in London, Robert Lamb completed his undergraduate and doctor of philosophy degrees at the Universities of Birmingham and Cambridge, respectively. After receiving his Ph.D. in virology, Lamb secured a post-doctoral research associate position at the prestigious Rockefeller University in New York, under the tutelage of Dr. Purnell...
Choppin. A preeminent researcher in the field of virology, Choppin would go on to become the President of the Howard Hughes Medical Institute (HHMI) from 1987-1999. During his tenure, the organization experienced one of the largest growth spurts in HHMI’s history, including an expansion of the number of investigators from 96 to 330. Dr. Lamb later joined this group of leading researchers in 1991.

Indeed, Lamb attributes much of his success in the laboratory and beyond to the generous support given by HHMI:

"The Howard Hughes Medical Institute has provided me with the opportunity to take on high-risk research that would not be at all possible with government funding. [HHMI] invests in the people, not their research, so that they are not afraid to take risks."

Government grants for biomedical research, offered through the Public Health Service, often require much of the research to be previously completed with nearly certain guarantee of success. Crystallography and subsequent visualization of protein structure, on the other hand, are risky and uncertain procedures that often result in failure. Without the investment by HHMI, Professor Lamb’s laboratory would not have been able to successfully solve the crystal structure of the fusion protein from the human paramyxovirus.

Despite many accomplishments in the laboratory, Robert Lamb is not a scientist to restrict himself to the bench. In addition to his work under the auspices of the program for Biochemistry, Molecular Biology and Cell Biology, Lamb enjoys a joint appointment with the Northwestern University Feinberg School of Medicine in the Department of Immunology. He has also previously served as the Chair of the CDC Influenza Virus Branch External Review Committee, on the Board of Scientific Counselors of the National Institute for Allergies and Infectious Disease, and on the Director’s Review Group for the CDC. In this fashion, Lamb seamlessly transfers his extensive biomedical knowledge to the realm of public health policy, making him a renaissance scientist who recognizes the important interdisciplinary applications of basic biomedical research.

Space filling three-dimensional model of the RSV core fusion protein.
Dr. Richard Silverman, the Charles Deering McCormick Professor of chemistry, is active and accomplished both as a lecturer and a researcher, teaching several quarters from the organic chemistry sequence for majors and the popular advanced course "Medicinal Chemistry: The Organic Chemistry of Drug Design and Action." But Professor Silverman is best known for his efforts behind Lyrica™, marketed by Pfizer Pharmaceuticals for the treatment of neuropathic pain and partial epileptic seizures. Approved by the European Union's Committee for Proprietary Medicinal Products and on the shelf in the UK, Lyrica™ won approval by the Federal Drug Administration this past December.

The research performed in the Silverman group that pioneered Lyrica™ is centered on the interactions of enzymes and their molecular substrates. Professor Silverman's students investigate the molecular mechanisms of action and the rational design and synthesis of potential therapeutic agents. Thus, "You work with the idea that if you select the right enzyme it might be important to a disease state," and after the potential enzyme is identified, its particular molecular mechanism of action is developed using chemical model studies. And since many drugs act as inhibitors of enzymes, potential candidates are obtained through organic synthesis and tested for activity. The rational design of the drug is the most important step, and is developed using a combination of data gathered from X-Ray crystallography, MALDI-TOF and ESI mass spectrometry and radiolabelling studies detailing interactions of the molecules with the active site of the enzyme, the principles of which Professor Silverman teaches in his medicinal chemistry course.

Some of the enzymes studied by the Silverman group are involved in neurodegenerative disorders, research into which led to Lyrica™, the brand name of pregabalin. Epilepsy in many cases is due to an imbalance between the inhibitory neurotransmitter gamma-aminobutyric acid (GABA) and the excitatory neurotransmitter glutamate. If levels of GABA drop, too much excitatory neurotransmission occurs, leading to convulsions. The molecule GABA is degraded by GABA aminotransferase and synthesized via glutamate decarboxylase. Thus finding an inhibitor of GABA aminotransferase or an activator of glutamate decarboxylase could potentially manage the imbalance. Pregabalin was originally designed as a GABA aminotransferase inhibitor, but which in fact acts by blocking a calcium ion channel that re-
leases glutamate, producing the same physiological effect. Furthermore, pregabalin proved successful in that it was capable of binding a protein transporter and crossing the blood-brain barrier, which is necessary for drug action. Professor Silverman is currently conducting structure-based design of new inhibitors and inactivators of brain GABA aminotransferase in collaboration with a crystallography group, using high resolution crystal structures of several potential drugs bound to GABA aminotransferase.

Another enzyme investigated by the Silverman group is nitric oxide synthase (NOS), the enzyme that generates the important cellular second messenger nitric oxide. This enzyme is known to exist in three isozymic forms: nNOS which is found in the brain, inducible iNOS found in macrophages and eNOS found in endothelial cells. Inhibiting nNOS may be important in treating neurotoxicity and stroke, but selective inhibition of nNOS is necessary to prevent disruption of nitric oxide signaling elsewhere in the body. New classes of inhibitors selective for nNOS are currently under investigation.

The Silverman group also is interested in the synthesis of cyclic peptides and cyclic depsipeptides as potential inhibitors for two enzymes, histone deacetylase and topoisomerase I, which are important to tumor cell growth. Professor Silverman is also interested in designing inhibitors of enzymes in the mevalonate pathway of _S. pneumoniae_ as selective antimicrobial agents.

Apart from training graduate students, Professor Silverman is committed to undergraduate participation in research. He has mentored countless undergraduates in his lab, many of whom have pursued medicine and graduate study in chemistry. Professor Silverman currently has 213 publications and holds 37 patents. Dr. Silverman was recognized as a Fellow of the American Association for the Advancement of Science and a Fellow of the American Institute of Chemists and received the Arthur C. Cope Senior Scholar Award of the American Chemical Society as well as many teaching awards, culminating in the Northwestern University Alumni Teaching Award given in 2000.
An Error Analysis of the Phased Array Antenna Pointing Algorithm for STARS Flight Demonstration #2

Michael P. Carney
DEPARTMENT OF ELECTRICAL ENGINEERING

James C. Simpson
MENTOR, LEAD: RANGE SYSTEMS AND DEVELOPMENT BRANCH,
NASA KENNEDY SPACE CENTER, FLORIDA

Introduction
Space-Based Telemetry and Range Safety (STARS) is a multicenter National Aeronautics and Space Administration (NASA) project to determine the feasibility of using space-based assets, such as the Tracking and Data Relay Satellite System (TDRSS) and Global Positioning System (GPS), to increase flexibility (e.g., increase the number of possible launch locations and manage simultaneous operations) and to reduce operational costs by decreasing the need for ground-based range assets and infrastructure. When a rocket is launched, a series of ground stations monitors its progress into space for the first 8.5 minutes of its flight. These ground stations use radar, which has far less range than TDRS and GPS. Therefore, multiple ground stations are required to monitor the flight, which leads to high maintenance costs. These costs would be greatly reduced through the use of STARS equipment; essentially, the launch vehicle will monitor its own position and attitude and transmit it through TDRS to a single ground station. The STARS project includes two major systems: the Range Safety and Range User systems. The latter system uses broadband communications (125 kbps to 500 kbps) for voice, video, and vehicle/payload data. Flight Demonstration #1 revealed the need to increase the data rate of the Range User system. During Flight Demonstration #2, a Ku-band phased-array steerable antenna will generate a higher data rate and will be designed with an embedded pointing algorithm to guarantee that the antenna is pointed directly at TDRS. This algorithm will utilize the onboard position and attitude data to point the antenna to TDRS within a two-degree full-angle beamwidth. This report investigates how errors in aircraft position and attitude, along with errors in satellite position, propagate into the overall pointing vector.
Explanation of Variables between the Aircraft and TDRS

The antenna on the aircraft must have a direct line-of-sight to TDRS, which is represented by the blue arrow in Figure 1. The vector is defined by azimuth ($\alpha$) and elevation ($\varepsilon$) angles in the antenna platform coordinate system. The antenna's actual pointing vector, represented by the red arrow and angles $\alpha'$ and $\varepsilon'$, might differ from the desired vector pointing to TDRS. The angle $\delta$ represents the total angular difference between TDRS vector and the antenna pointing vector. To prevent inaccuracies, the antenna must have less than a $2^\circ$ full-angle beamwidth in order to view TDRS.

Coordinate Systems and Transformations

TDRS satellites are nearly geostationary in low-inclination orbits with known position represented as latitude, longitude, and altitude.

The aircraft's position is represented in the Earth-Centered Earth-fixed (ECEF) coordinate system with its attitude position defined by heading, pitch, and roll angles. The ECEF coordinate system has three axes labeled $E$, $F$, and $G$. The $G$-axis points from the center of the earth to the north pole (along the axis of rotation); the $E$-axis points from the center of the earth to the equator at the prime meridian; and the $F$-axis is derived from the $E$- and $G$-axes to be orthogonal, pointing into the Indian Ocean at the equator. See Figure 2.

In order to generate the azimuth and elevation of the aircraft depicted in Figure 1, the TDRS latitude, longitude, and altitude, is first transformed into the ECEF coordinate system. This is necessary to get an accurate pointing vector to TDRS from the aircraft. Currently, we have the aircraft's ECEF position, so performing this transformation will generate TDRS coordinates in the same coordinate system. If $R_1$ is the distance from TDRS to Earth center for geosynchronous range, and $\varphi_1$ and $\lambda_1$ represent TDRS latitude and longitude, respectively, then

$$\vec{R}_{EG} = \begin{pmatrix} R_1 \cos \varphi_1 \cos \lambda_1 \\ R_1 \cos \varphi_1 \sin \lambda_1 \\ R_1 \sin \varphi_1 \end{pmatrix}$$

These three values represent the $E$, $F$, and $G$ coordinates of TDRSS, respectively. The first pointing vector can be found by subtracting the aircraft's ECEF coordinates from the TDRS ECEF coordinates. The goal in performing the coordinate transformations is to be able to solve for exact azimuth and elevation angles between the aircraft and TDRS. Without this vector, the pointing algorithm would not be able to point the antenna correctly.
The latitude, longitude, and altitude of the aircraft are then required to complete the first coordinate transformation. If \( \varphi, \lambda, \) and \( h \) are the geodetic latitude, longitude and altitude, respectively, of the aircraft, and \( E, F, \) and \( G \) are the GPS ECEF coordinates, then

\[
\varphi = \tan^{-1}\left( \frac{G}{\sqrt{E^2 + F^2}} \right)
\]

\[
\lambda = \tan^{-1}\left( \frac{F}{E} \right)
\]

\[
h = \sqrt{E^2 + F^2 + G^2} - a \sqrt{1 - e^2 \sin^2 \varphi}
\]

where the following WGS-84 values were used:

\[
\xi = 1.006794229 = a^2 / b^2
\]

\( a \) = Earth equatorial radius = 6378137 m

\( b \) = Earth polar radius = 6356752.3142 m

\( e^2 \) = eccentricity squared = 0.00669437999014132

This vector can now be transformed from the ECEF coordinate system to the Local Horizontal Coordinate System. This coordinate system is also called the earth-fixed axis or the geographic coordinate system. One axis points to the north, one to the east, and one axis is either up or down. See Figure 3. For this application we will be using the north-east-up method (NEU). The local horizontal system is related to the ECEF system through latitude, longitude, and altitude.

Since \( \vec{R}_{EFG} \) and \( \varphi, \lambda \) were previously defined, the NEU position vector is:

\[
\vec{R}_{NEU} = \Phi \vec{R}_{EFG}
\]

where

\[
\Phi = \begin{pmatrix}
-\cos \lambda & -\sin \lambda & \cos \varphi_A \\
\sin \lambda & \cos \lambda & 0 \\
\cos \lambda & \cos \varphi_A & \sin \lambda & \cos \varphi_A & \sin \varphi_A
\end{pmatrix}
\]

The NEU position vector coordinates, found above, are then rotated through the heading, pitch, and roll angles of the aircraft. The new coordinates formed by this rotation are in the Body-Axis coordinate system, also known as the body platform coordinate system. It is a right-handed system with \( x \)-axis forward, \( y \)-axis to the right, and \( z \)-axis pointed down. See Figure 4. This coordinate system is related to the local horizontal system through the heading, pitch, and roll angles of the aircraft. The azimuth and elevation cannot be found without integrating the attitude data of the aircraft, and this transformation accomplishes this.

\[\text{Figure 3. Local Horizontal Coordinate System Diagram.}\]

\[\text{Figure 4. Body-Axis Coordinate System Diagram.}\]
\( \vec{R}_{\text{NEU}} \) was defined above, and if \( H, P, \) and \( R \) correspond to the aircraft’s heading, pitch, and roll, respectively, then

\[
\vec{R}_{\text{xyz}} = \Gamma \vec{R}_{\text{NEU}} = (\Gamma_H \Gamma_P \Gamma_R) \vec{R}_{\text{NEU}}
\]

where

\[
\Gamma_H = \begin{pmatrix} \cos H & \sin H & 0 \\ -\sin H & \cos H & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Gamma_P = \begin{pmatrix} \cos P & 0 & \sin P \\ 0 & 1 & 0 \\ -\sin P & 0 & \cos P \end{pmatrix}, \quad \Gamma_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos R & -\sin R \\ 0 & \sin R & \cos R \end{pmatrix},
\]

and

\[
\Gamma = \begin{pmatrix} \cos H \cos P & \cos P \sin H & \sin P \\ -\cos R \sin H + \cos H \sin P \sin R & \cos H \cos R + \sin H \sin P \sin R & -\cos P \sin R \\ -\cos H \cos R \sin P - \sin H \sin R & -\cos H \sin R \sin P + \cos H \sin R & \cos P \cos R \end{pmatrix}.
\]

The order of rotation is of particular importance. The final azimuth and elevation angles are then calculated from the \( \vec{R}_{\text{xyz}} \) vector.

**Azimuth:** \( \alpha = \tan^{-1} \left( \frac{Y}{X} \right) \)

**Elevation:** \( \phi = \tan^{-1} \left( \frac{Z}{\sqrt{X^2 + Y^2}} \right) \)

### Comparing the Azimuth and Elevation with the CLASS Results

The above calculations for azimuth and elevation derived from rotations and transformations were compared with the pointing angles for the Range Safety system obtained from the Communications Link Analysis and Simulation System (CLASS) at Goddard Space Flight Center for the last two flights of STARs Flight Demonstration #1.

These flights were chosen because they were representative of the flight profiles flown during Flight Demonstration #1: Flight 6 was a low dynamic flight and Flight 7 was highly dynamic (see Table 1).

CLASS used a body-axis coordinate system defined as follows:

- \( \theta = 0^\circ, \quad \phi = 0^\circ \) is straight up
- \( \theta = 90^\circ, \quad \phi = 0^\circ \) is straight down
- \( \theta = 90^\circ, \quad \phi = 90^\circ \) is out the starboard wing
- \( \theta = 90^\circ, \quad \phi = 180^\circ \) is out the port wing
- \( \theta = 0^\circ, \quad \phi = 270^\circ \) is out the port wing

### Table 1. Flight Profiles

<table>
<thead>
<tr>
<th>Flight</th>
<th>Date</th>
<th>Maneuvers</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>July 9, 2003</td>
<td>Long Distance (Level Flight Mach 0.85 at 20k ft &amp; 15k ft)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Straight &amp; Level (Turns)</td>
</tr>
<tr>
<td>7</td>
<td>July 15, 2003</td>
<td>Dynamic (Rolls, Loops, POUs)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Launch Vehicle Simulation (45(^\circ) ascent to 30k ft &amp; 70(^\circ) descent from 30k ft)</td>
</tr>
</tbody>
</table>
It is important to note that this coordinate system is for the far-field where the plane can be considered as a point-mass; consequently, there is no explicit origin for this coordinate system. Each value of $\theta$ defines a new set of $\phi$ values.

(Note: Throughout the STARS project, $\phi$ has been used in this way to define the antenna coordinate system. In the earlier sections of this paper, $\phi$ also represented the geodetic latitude, as is the usual custom. The geodetic latitude was subscripted to prevent confusion. The author apologizes for this, but decided not to change the symbols just for this section of the paper.)

Let $\mathbf{R}'$ be the vector from the center of the aircraft to the observation point, e.g. a TDRSS satellite. Drop a perpendicular line segment from $\mathbf{R}'$ onto the x-axis (Figure 5a). Use this location on the x-axis as the origin for a new set of phi angles as defined above (see Figure 5b).

As shown by the diagrams,

$$\begin{align*}
x &= R' \cos(\theta) \\
z' &= R' \sin(\theta)
\end{align*}$$

and

$$\begin{align*}
z &= z' \cos(\phi) = R' \sin(\theta) \cos(\phi) \\
y &= z' \sin(\phi) = R' \sin(\theta) \sin(\phi)
\end{align*}$$

The final azimuth and elevation calculations can then be derived. Figure 6a and Figure 6b below show the graphical derivation for the azimuth and elevation calculations, respectively.

The azimuth is then calculated:

$$\tan \alpha = \frac{y}{x} = \frac{R' \sin(\theta) \sin(\phi)}{R' \cos(\theta)} = \frac{\sin(\theta) \sin(\phi)}{\cos(\theta)}$$

$$\alpha = \tan^{-1} \left( \frac{\sin(\theta) \sin(\phi)}{\cos(\theta)} \right)$$

The elevation is then calculated:

$$\tan \epsilon = \frac{z}{\sqrt{x^2 + y^2}} = \frac{R' \sin(\theta) \cos(\phi)}{\sqrt{R'^2 \cos^2(\theta) + R'^2 \sin^2(\theta) \sin^2(\phi)}}$$

$$\epsilon = \tan^{-1} \left( \frac{\sin(\theta) \cos(\phi)}{\sqrt{\cos^2(\theta) + \sin^2(\theta) \sin^2(\phi)}} \right)$$

Figures 5a and 5b. Azimuth Calculation Diagram.

Figures 5a, 5b, 6a, 6b. Elevation Calculation Diagram.
The two methods in calculating azimuth and elevation were then plotted against each other in Figure 7 and Figure 8 below.

The plots show great similarity between the pointing algorithm and CLASS results. Notable factors affecting slight differences include a slight offset due to a timing difference of 10s during Flight 6 and a discontinuity in azimuth at 0° and 360°.

**Errors Added into the Algorithm**

After confirming the azimuth and elevation values, errors were added into the position and attitude of the aircraft at each time increment. These perturbed values were subtracted from the unperturbed values of azimuth and elevation ($\alpha$ and $\phi$, respectively) to form new perturbed values of azimuth and elevation, represented by $\alpha'$ and $\phi'$, respectively. This was done to see how the pointing algorithm would react to imperfect data. In addition, the overall angular difference between the antenna's pointing vector and the TDRS pointing vector, $\delta$, was calculated using the spherical law of cosines,

$$
\hat{e}_{\text{TDRS}} \cdot \hat{e}_{\text{ANT}} = (1)(1) \cos \delta = \sin \phi \sin \phi' + \cos \phi \cos \phi' \cos (\alpha - \alpha')
$$

where $\hat{e}_{\text{TDRS}}$ is a unit vector along the true direction from the antenna to TDRS, and $\hat{e}_{\text{ANT}}$ is a unit vector in the direction the antenna is pointing. These two vectors are also displayed graphically in Figure 1.
Results
One hundred and sixty-four trials were performed with various combinations of perturbations in the aircraft’s location and attitude and in the TDRS location. Aircraft position errors seemed to have a reduced effect on the data than did changes in aircraft attitude or TDRS position. A one-degree change in heading, pitch, or roll created significant differences in azimuth and elevation delta values, but even a 10,000 meter change in aircraft position had little effect. Changes in TDRS position had an even larger effect on the azimuth and elevation delta values. See Table 2 for full results.

Without the antenna pointing directly at TDRS, no data can be exchanged between the aircraft and the ground station. Without data exchange, there would be no reason to use space-based assets. Without seeing the extent to which the data can be perturbed while still transmitting and receiving data with the aircraft, the full ability of the antenna pointing algorithm will not be realized.

When attitude error was introduced without any position error, the maximum error found by the pointing algorithm was nearly identical to the root-sum-square of the attitude error. Ten random combinations of attitude perturbations were used, and the total angular difference was within two percent of the root-sum-square of the individual attitude angular errors. There is an approximately one-to-one correlation between TDRS position errors and the total angular difference.

Change in TDRS position had an even greater effect on the overall angular difference, with a 0.05 degree change in latitude or longitude producing an unreasonable δ (see Figure 9). The numbers next to

Table 2. Summary of Results

<table>
<thead>
<tr>
<th>Flight</th>
<th>Tot. Trial</th>
<th>Quantity Perturbed</th>
<th>Size of Perturbation</th>
<th>Elevation Error (°)</th>
<th>Azimuth Error (°)</th>
<th>Delta (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>10</td>
<td>E (aircraft)</td>
<td>-10km → 10km</td>
<td>-0.1 → 0.1</td>
<td>-0.1 → 0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>F (aircraft)</td>
<td>-10km → 10km</td>
<td>-0.05 → 0.05</td>
<td>-0.08 → 0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>G (aircraft)</td>
<td>-10km → 10km</td>
<td>-0.05 → 0.05</td>
<td>-0.05 → 0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>H (heading)</td>
<td>20° → 20°</td>
<td>-10 → 10</td>
<td>-28 → 28</td>
<td>19.2 ± 0.1</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>P (pitch)</td>
<td>20° → 20°</td>
<td>-20 → 20</td>
<td>-12 → 12</td>
<td>5.1 → 20</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>R (roll)</td>
<td>20° → 20°</td>
<td>-20 → 20</td>
<td>-17 → 17</td>
<td>2 → 20</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>TDRS Lat.</td>
<td>2° → 2°</td>
<td>-2 → 2</td>
<td>-2.5 → 2.5</td>
<td>2.1</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>TDRS Lon.</td>
<td>2° → 2°</td>
<td>-2 → 2</td>
<td>-2.5 → 2.5</td>
<td>2.1</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>E (aircraft)</td>
<td>-10km → 10km</td>
<td>-0.1 → 0.1</td>
<td>-8.9 → 7.6</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>F (aircraft)</td>
<td>-10km → 10km</td>
<td>-0.06 → 0.06</td>
<td>-3.9 → 3.4</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>G (aircraft)</td>
<td>-10km → 10km</td>
<td>-0.05 → 0.05</td>
<td>-3.0 → 3.3</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>H (heading)</td>
<td>20° → 20°</td>
<td>-19.2 → 19.2</td>
<td>-175 → 180</td>
<td>19.2 ± 0.1</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>P (pitch)</td>
<td>20° → 20°</td>
<td>-20 → 20</td>
<td>-171 → 169</td>
<td>5.33 → 20</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>R (roll)</td>
<td>20° → 20°</td>
<td>-20 → 20</td>
<td>-180 → 180</td>
<td>0.01 → 20</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>TDRS Lat.</td>
<td>1° → 1°</td>
<td>-2.1 → 2.1</td>
<td>most ± 10°, some ± 50°</td>
<td>2.1</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>TDRS Lon.</td>
<td>1° → 1°</td>
<td>-2.1 → 2.1</td>
<td>most ± 10°, some ± 50°</td>
<td>2.1</td>
</tr>
</tbody>
</table>
the title of the plot signifies the error in the following order: x-position / y-position / z-position / heading-angle / pitch-angle / roll-angle. The minima and maxima of \( \delta \), the times at which they occurred, the mean, and the standard deviation of \( \delta \) are also given.

The data reveals some very important details of the pointing algorithm. Flight 7 was much more dynamic than Flight 6, and the errors are greater for Flight 7 overall. This is especially true for the elevation error when the heading value was perturbed. The error increased by nearly 100% from Flight 6 to Flight 7. Additionally, azimuth error increased substantially from Flight 6 to Flight 7 for both aircraft position and attitude. This makes sense, since the high dynamics of Flight 7 would cause the these two values to constantly change at high rates and would require a greater precision by the antenna. Figure 9 below includes a dotted line at 2-degrees to distinguish between acceptable and unacceptable errors.

The error analysis performed agrees with the expected results. A positional change should not have a large effect on the pointing of the antenna, due to the large distance between the aircraft and TDRS. A change in attitude, however, should have a somewhat greater effect since it alters the precise azimuth and elevation angles that were so carefully calculated previously. It also follows that a high-dynamic flight, such as in Flight 7, would produce greater errors than those of less-dynamic flight. The greater the frequency of movement by the aircraft, the less time the antenna will have to find an exact pointing vector to TDRS.
Characterizing Cell Motility of Cancerous Neuronal and Progenitor Cells

Kwan Y. Chen  
DEPARTMENT OF BIOMEDICAL ENGINEERING

Anjen Chenn  
FACULTY ADVISOR, PATHOLOGY, FEINBERG SCHOOL OF MEDICINE, NORTHWESTERN UNIVERSITY

ABSTRACT

Glioblastomas are one of the most malignant types of brain tumors and are characterized by their ability to rapidly metastasize and migrate to nearby normal tissues. We are studying how U-118 MG and T98G glioblastoma cell lines migrate in vitro on three commonly found ECM molecules, collagen, fibronectin, and laminin, with a novel phagokinetic migration assay. In this assay, cells phagocytize and displace 1 μm diameter fluorescent beads coated on 6-well plates. Images are taken by fluorescent microscopy after the cells are allowed to migrate for eighteen or twenty-four hours, and the area cleared by each randomly selected single cell is determined and analyzed. The results indicate that the ability of cells to migrate depends on the type of substrate. Our assay demonstrates that U-118 MG cells are significantly more motile on collagen whereas T98G cells do not have a particular substrate preference for migration. This suggests that the cell lines differ in the type of integrins, proteins responsible for cell-ECM adhesion, found on the cell membrane.

I. INTRODUCTION

The cerebral cortex is the center of our highest cognitive and intellectual functions. During development, immature cerebral cortical neurons migrate long distances from where they are generated to their final position. Problems in neuronal migration are thought to underlie a number of serious and devastating neurological disorders such as epilepsy, lissencephaly (malformation of the brain), microcephaly (failure of brain growth), and mental retardation.

Cerebral cortical neurons are generated in a region of progenitor cells called the ventricular zone. Neurons are thought to migrate by following fibers from specialized guiding cells called radial glial cells from the ventricular zone to their final positions in the cerebral cortex. However, not all cortical neurons
migrate in such an orderly fashion; cancerous cells in particular migrate away from the neoplasm through an ECM layer composed of molecules such as fibronectin, collagen, and laminin to colonize other parts of the brain. It is important to study cell motility and its relationship to tumor malignancy since their rapid metastasis makes surgical removal of such tumors difficult.

Cell migration occurs through a series of steps, involving the protrusion of filopodia and lamellipodia, attachment to a substrate, traction of the cell towards the filopodia, along with the simultaneous retraction of the opposite end of the cell. By using a phagokinetic assay with varying ECM molecules, we are able to visually and quantitatively determine the importance of the substrate in cell adhesion during cell migration.

We are using a phagokinetic assay to study the roles that ECM molecules fibronectin, laminin, and collagen play in cell motility and tumor malignancy of two human glioblastoma cell lines, U-118 MG and T98G. In this assay, 1 μm diameter fluorescent beads are coated on 6-well plates and allowed to adhere to the bottom of each well. Cells are then sparsely plated on each well. These cells are allowed to migrate and phagocytize the fluorescent beads in each well for a specified time before being fixed. As the cell migrates, it leaves behind a negative track. We can then determine the area migrated as an estimate of how motile the cell is. The U-118 MG is considered a tumor grade III glioblastoma while the T98G is a grade IV. Because the T98G cells are of higher grade, we expected that these cells would be more motile than the U-118 MG.

II. METHOD

We assessed cell motility of our glioblastoma cell lines with a phagokinetic assay. 2 mL of 0.05% solution of fluorescent green carboxylate-modified Fluospheres (Molecular Probes) mixed with phosphate buffer solution (PBS) were added to each well of poly D-lysine coated 6-well plates. Each well has a diameter of 3.5 cm. The plate was incubated at room temperature for two hours for the Fluospheres to settle to the bottom of each well. 0.25% trypsin-1mM EDTA was used to remove primary astrocytes derived from embryonic mouse brains, U-118 MG, and T98G cell lines from cell-culture flasks. The cells were centrifuged and resuspended with Dulbecco's Modified Eagle Medium and 20% fetal bovine serum at a density of 2000 cells/mL. This low concentration of cells required passing the suspension through a cell strainer with 8-μm pores to separate cell aggregates. The excess Fluosphere solution from each well was removed and gently rinsed with PBS. Finally, 2 mL (~4000 cells) of the cell suspension were dispensed into each well. The cells were incubated in 5% CO₂ and 37 degrees Celsius for 18 hours. The experiment was repeated with laminin, fibronectin, collagen, and poly D-lysine coated 6-well plates with an incubation time of 24 hours.

After incubation, the cells were fixed with 4% paraformaldehyde for 20 minutes. The cell nuclei and actin filaments were fluorescently labeled with Hoescht 33342 (Sigma) and Alexa-Fluor 546 Phalloidin (Molecular Probes), respectively. The cells were observed with a Nikon TE2000 microscope with a 10× objective lens.

To analyze how much area each cell had cleared, we used a 13×13 transparent square grid with each side equal to the 3.5 cm diameter of each circular well to help us divide the well into squares of 10× field-of-view. Images of randomly chosen 10× field-of-views were captured with RT Slider CCD camera from SPOT Systems. At least 60 individual cells were identified for each experimental condition. The area cleared for each cell line was determined with ImageJ version 1.32 from the National Institutes of Health. Statistical analysis was performed with Matlab 7.0.

III. RESULTS

For the first experiment, we used a poly D-lysine-coated 6-well plate and incubated the cells for 18 hours after plating them onto each well. It can be relatively easy to visually assess the differences in cell motility between the primary astrocytes (Figure 1), U-118 MG (Figure 2), and T98G (Figure 3) cells. Using one-factor analysis of variance (ANOVA), we determined that the areas cleared by these cells were significantly
Figure 1. Cell motility of primary astrocytes. Primary astrocytes were plated for 18 hours at a density of 4000 cells/well on a 6-well plate coated with poly D-lysine and covered with 1 μm fluorescent beads (green). The cells have been stained for actin with phalloidin (red) and cell nuclei with Hoescht (blue). As the cells move, it leaves behind dark tracks as the cells phagocytize the fluorescent beads.

Figure 2. Cell motility of U-118 MG glioblastoma cell line. The U-118 MG cells were plated on the phagokinetic assay for 18 hours under similar conditions as the primary astrocytes (Figure 1) and T98G (Figure 3).

Figure 3. Cell motility of T98G glioblastoma cell line. The T98G cells were plated on the phagokinetic assay for 18 hours.
different from each other \( F(\text{df}=2) = 34.28, p = 3.36 \times 10^{-11}, p < 0.05 \) with the U-118 MG cells clearing 6663 ± 269.6 \( \mu \)m², T98G cells clearing 2677 ± 832 \( \mu \)m², and primary astrocytes clearing 649 ± 270 \( \mu \)m² (\( \mu \pm \text{SEM} \)) in 18 hours.

When the incubation time was increased to 24 hours, greater significance between the U-118 MG and the other cell lines were observed (Figure 4). Another one-factor ANOVA was performed with results similar to those from the 18-hour condition. All three cell lines exhibited a significant difference in cell motility \( F(\text{df}=2) = 110.4, p = 4.24 \times 10^{-34}, p < 0.05 \) with the U-118 MG cells clearing 19462 ± 1443.8 \( \mu \)m², T98G clearing 6437 ± 718.5 \( \mu \)m², and primary astrocytes clearing 2273 ± 220.5 \( \mu \)m² (\( \mu \pm \text{SEM} \)) in 24 hours.

We also ran the 24-hour experiment with laminin-, collagen-, and fibronectin-coated 6-well plates (Figure 5). A two-factor ANOVA was performed to analyze the data, and similar to the 18-hour experiment performed with only poly D-lysine, we observed significant clearing differences between cell lines \( F(\text{df}=2) = 42.64, p = 2.84 \times 10^{-4}, p < 0.05 \).

While the type of substrate did not impact the motility of the cell lines to the same degree, we did notice that primary astrocytes plated on poly D-lysine were significantly less motile than those plated on laminin, collagen, or fibronectin extracellular molecules according to the Tukey-Kramer comparison procedure at \( \alpha = 0.05 \). Primary astrocytes plated on collagen migrated significantly faster than on other substrates while cells plated on poly D-lysine were significantly less motile than those on laminin, fibronectin, or collagen. Significant differences in T98G cells were observed when placed on fibronectin and poly D-lysine. Finally, U-118 MG cells were significantly more motile on collagen than on laminin, fibronectin, or poly D-lysine.

IV. DISCUSSION

Extracellular matrix (ECM) components support the migration of a wide variety of cell types. The ECM is composed of a heterogeneous mix of secreted molecules. The distribution of
ECM molecules has been characterized in the developing cerebral cortex and a number of these molecules have shown that they play key roles in cell signaling for migration and proliferation functions. Laminin, fibronectin, and collagen are major ECM proteins found in the perivascular region of the adult brain, which is a main pathway where gliomas migrate.

Previous phagokinetic migration assays used gold particles on various cell types to examine cell motility. However, we found 1 μm diameter Fluospheres more suitable for this study because the Fluospheres were easier to locate inside a cell than gold particles. We did not observe any side-effects of the Fluospheres inside the cells. Cells both displace and ingest the Fluospheres, generating tracks that can be visualized by fluorescent microscopy and later measured.

After the cells were plated onto the substrates and incubated in each well for 18 hours, we were able to detect significantly different cell motilities between the three cell types. In our experiments, primary astrocytes were used as a control because these cells are not cancerous nor considered as motile as U-118 MG or T98G cells; we observed that primary astrocytes cleared the least amount of area compared to the other cell lines. In addition, we observed little difference in motility of primary astrocytes plated on fibronectin, collagen, and laminin, but a significant difference between these three ECM molecules compared to poly D-lysine. Since poly D-lysine is simply a thin layer of amino acids, this confirms that ECM molecules play an extremely important aspect in cell motility of non-cancerous primary astrocytes.

Our experiments also demonstrated that U-118 MG cells cleared the largest area, and T98G cells cleared an average area between that of the U-118 MG and primary astrocytes. These results obtained from the 18-hour experiment were consistent with the results from the 24-hour experiment with the exception that we observed a lower variability in the 24-hour experiment. We had initially expected T98G cells to be more motile that U-118 MG because they are classified as a higher tumor grade, but did not observe this. This observation suggests that in addition to cell motility, the rate of cell proliferation is also important in determining the malignancy of a glioblastoma cell line. We have noticed that T98G cells do grow more rapidly than U-118 MG cells in cell culture flasks but have not performed any assays to assess this.

Through our two-factor ANOVA, we were able to clearly differentiate how well primary astrocytes, U-118 MG, and T98G cells migrate on our phagokinetic assay. The extracellular substrates were then compared within each cell type using one-factor ANOVA and Tukey-Kramer comparison procedures. Some studies have shown that collagen substrates promote higher motility in high tumor grades and lower motility in lower tumor grades, which is inconsistent with our results. Rather, our experiments demonstrate that the U-118 MG cells, which are of lower tumor grade, are significantly more motile than T98G cells on collagen in a phagokinetic assay. Because none of the T98G cells plated on the three ECM molecules were significantly different, while U-118 MG cells are significantly more motile on collagen, we believe that the composition of molecules on the cell membrane of T98G and U-118 MG cells are different.

The family of structural proteins known to anchor the cell to the ECM is integrins. Integrins are composed of heterodimers, with αβ1, αβ1, αβ8, αβ1, αβ3, and αβ5 integrins already shown to play roles in cell migration by adhering to the ECM molecules and activating migratory signal transduction cascades. Specifically, α3, α1, β1, and β3 subunits have previously been identified in glioblastoma multiforme at higher levels than normal brain tissues. Many of these migratory signal transduction cascades are mediated through the integrin-linked kinase (ILK) before activating mitogenic pathways. The lack of a single substrate in our experiment that severely enhances or interferes with cell migration of T98G and U-118 MG cells indicates that different integrin subunits are expressed on the cell membranes of these two cell lines.

It must be noted that there are aspects in our in vitro experiments that do not naturally occur in vivo. For example, our plates are coated with either no ECM molecules or only one
ECM molecule at a time. In the human brain however, these cells would often be surrounded by a mixture of different ECM molecules. It has been observed in literature that different integrin subunits regulate signal transduction with each other, or that multiple integrin complexes can bind to the same ECM molecules. Perhaps plating cells on a mixture of ECM molecules instead of just one type of ECM molecules can further enhance cell motility. To determine if integrins are the major cause of differences in cell motilities between U-118 MG, T98G, and primary astrocytes, we can determine the expression levels of the integrin subunits, and determine the affinity of these subunits to fibronectin, laminin, and collagen.

Our experiments demonstrate that the motility of glioblastoma cells are often dependent on the cell line. In addition, the substrate the cell interacts with can increase or decrease cell motility, although from our experiments, this is also dependent on the cell line. Because the malignancy of a tumor is often related to how rapidly the cancerous cells metastasize, understanding cell migration will allow researchers to devise drugs to prevent cells from migrating. In addition to allowing more effective surgical removal of tumor cells, reducing cell motility will slow down the rate of cancer cells from metastasizing and encouraging the use of less invasive therapies.

REFERENCES


The Physics and Applications of Index Patterned Fabry-Pérot Lasers

James M. Rondinelli  
DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING

Eoin O'Reilly  
PRINCIPAL INVESTIGATOR, PHOTONIC RESEARCH GROUP, TYNDALL INSTITUTE, CORK, IRELAND

Stephen O'Brien  
POST-DOCTORAL RESEARCHER, PHOTONIC RESEARCH GROUP, TYNDALL INSTITUTE, CORK, IRELAND

ABSTRACT

Typically, edge-emitting semiconductor Fabry-Pérot (FP) lasers operate multimode, though single-frequency sources are the most desirable for applications. FP lasers are simple to manufacture and with a recently developed method of patterning the effective refractive index seen by an optical mode propagating in the laser cavity, the spectral purity can be significantly improved. Using a transmission matrix approach to first order in the effective refractive index step the threshold condition is derived with respect to the cavity modes. This research extends that understanding to more complex structures and longer wavelengths. An Index Patterned Fabry-Pérot (IPFP) operating in the near infrared providing a comb of modes has been designed which offers useful prospects for multi-mode and tunable devices in telecommunications applications. IPFP lasers with increased spectral purity operating in the terahertz region of the electromagnetic spectrum have also been designed, providing an alternative to the present quantum cascade DFB devices. Ultimately, this work attempted to achieve a tunable laser source utilizing coupled cavities with IPFP sections. These IPFP lasers can be used to fabricate single-frequency communications lasers in a regime intermediate between plain FP lasers and those incorporating a periodic grating.

I. INTRODUCTION

Typically, edge-emitting semiconductor Fabry-Pérot (FP) lasers operate multimode, though single-frequency sources are the most desirable for applications. Classically, augmentation of the spectral purity of a semiconductor laser is achieved through a periodic variation of the refractive index, forming a grating within the laser cavity. This is the foundation upon which distributed feedback (DFB) lasers are designed. Fabrication of such lasers requires complex processing and the useful device yield can be poor, because of the presence of facet reflections with an essentially random phase with respect to the grating. Conversely, FP lasers are simple to manufacture and with a recently developed method of patterning the effective refractive index seen by an optical mode propagating in the laser cavity,
the spectral purity can be significantly improved. These Index Patterned Fabry-Pérot (IPFP) lasers incorporate index perturbations into the cavity which add a modulation.

This method requires minimal supplemental processing steps and involves the introduction of features in the laser ridge waveguide at the etching and lithographic stages. These perturbations take the form of slots with reflective characteristics that exhibit negligible changes to threshold current. Facet reflections form the fundamental feedback necessary for lasing while the effective index pattern provides enhanced spectral purity. The approach selects a particular cavity resonance; slots are added along the cavity to reduce the mirror losses of that mode. This research extends that understanding to more complex structures and longer wavelengths. An IPFP operating in the near infrared, providing a comb of modes, has been designed which offers useful prospects for multi-mode devices in telecommunications applications. IPFP lasers with increased spectral purity operating in the terahertz region of the electromagnetic (EM) spectrum have also been designed, providing an alternative to the complex quantum cascade (QC) DFB devices. In order to provide a foundation for review, a brief background on plain FP follows.

A Fabry-Pérot laser utilizes a standard cavity with reflecting mirrors at the boundaries to establish the cavity resonance. For the threshold condition to be achieved by a mode propagating in the cavity, the gain must be increased to compensate for the mirror losses, whereby the electric field replicates itself after a complete roundtrip in the cavity. Assuming that the effective refractive index is independent of frequency, the mirror losses in a FP laser will be equal for all modes. Consequently, the selectivity will only be derived from the wavelength dependency of the gain and the phase condition such that an integral number of half-wavelengths fit into the cavity or

\[ \frac{\lambda}{2n} = L \]  \hspace{1cm} (1.1)

where \( m \) is an integer mode, \( \lambda \) is the wavelength in free space, \( n \) is the refractive index of the cavity and \( L \) is the device length. Despite the central mode receiving the largest gain, if the mode separation is small enough, some side modes will also lase.

II. METHODOLOGY

II.1. Overview

Multimode sources are becoming more important as the desire to fabricate tunable lasers for advanced optical applications, such as wavelength division multiplexing networks, increases. Complex devices incorporating photonic crystals offer tunable DFB lasers in the near infrared regime. This work offers an alternative for the multimode source by designing an IPFP yielding a comb of modes which operate over the same frequency. Additionally, terahertz lasers are of great interest in a variety of applications, but development of narrow-band laser emission sources has been limited. By extension of the current method (described below), this regime is explored in an effort to determine the limits of the approach with regard to slot densities and index patterns. To achieve a spectrally pure laser emitting over this frequency requires complex processing which has
motivated the application of IFP devices. Work here has involved writing Mathematica® code that generates a pattern of slot positions to be introduced into the cavity. It seemed necessary to take a dual approach to this problem and divide the cavity at the center. Then slot positions could be generated to the right and to the left, independent of each other in order to reduce the mirror losses of a particular cavity resonance. Using FORTRAN, a series of programs were written which correct the slot positions generated above with respect to the optical path lengths.

II.2. Transmission Matrix Approach

The transmission matrix approach outlined below is used to determine the complex transmission coefficient of a multi-section cavity which has been deduced by S. O'Brien and E. P. O'Reilly. The method is restated here as insight into the novel engineering approach. For a multi-layer system, as in Figure 1, the complex transmission coefficient with one slot is found by considering a matrix product given by

$$\begin{align*}
\begin{bmatrix}
E_o
\end{bmatrix} &= T_{10} P_{1} T_{2} P_{2} T_{3} P_{3} T_{10} \begin{bmatrix}
\tilde{E}_o
\end{bmatrix} \\
&= \begin{bmatrix}
1 & r_0 \\
r_0 & 1
\end{bmatrix} \begin{bmatrix}
\tilde{E}_o
\end{bmatrix},
\end{align*}$$

where

$$T_{ij} = (t_{ij})^{-1} \begin{bmatrix}
1 & r_{ij} \\
r_{ij} & 1
\end{bmatrix},$$

and

$$P_{ij} = \begin{bmatrix}
\exp(-ik_{ij}L) & 0 \\
0 & \exp(ik_{ij}L)
\end{bmatrix}.$$
functions for integration and \( j = 1, 2, \ldots, N \). Final positions are found with corrections regarding the optical path lengths. With a known list of \( \delta_0 \), generated from Equation II.6, slot positions can be found, correcting for the optical path lengths, provided by \( \alpha_j = \Delta n / n \beta \). The phase requirement must also be satisfied so final slot positions are generated using

\[
\alpha_j = \frac{\eta_j + s_j \cdot \Delta n / n \beta}{1 + s \Delta n / n \beta}
\]  

(III.7)

where \( \alpha_j \) is the fraction of optical path length, \( \eta_j \) is the fractional cavity length, \( s_j \) is the number of slots to the left of slot \( j \) and \( \beta \) is the slot length as a fraction of cavity length. The mirror loss spectrum of the cavity can then be plotted. While trying to fill up the entire device with slots, convergence issues arose in creating the mirror loss plot. After investigation, it was found that the number or slots, \( s \) must be limited such that

\( \kappa L \ll 1 \), where \( \kappa \) may be defined as the average coupling coefficient for the device.

### III. RESULTS AND DISCUSSION

#### III.1. Device 1: The near infrared 5 mode comb

Here a near infrared device incorporating a 5-mode comb is presented. Through Fourier analysis and using a discrete number of slots, a mirror loss spectrum can be generated, with spacing, \( \alpha_0 \), cavity modes. The Fourier transform of the object function incorporates a series of Gaussian broadened delta functions, centered at the origin, with an equal spacing, \( \Delta m \), over \(-1/2 < \eta < 1/2\), where \( c_j = \eta_j - 1/2 \) and \( 0 < \eta_j < 1 \) is the fractional position of the slot center with respect to the cavity length. The cavity mode spacing is coupled to the parameter \( \tau \) which determines the magnitude of gain modulation at \( \Delta m = \pm \alpha \) modes by

\[
\tau^2 \leq \frac{1}{\pi \alpha^2} \ln(1 - \chi)
\]  

(III.1)

where \( \chi \) is the percent suppression desired. Equation III.1 is derived from the Gaussian envelope function, \( g(\Delta m) = \exp[-\pi \tau^2(\Delta m)^2] \), which is used to generate the object mirror loss spectrum, \( \Gamma(x) \) from Equation II.6. Here the mode spacing is taken as \( \alpha = 10 \) and \( \tau = 0.0057 \) providing for 1% suppression in mirror loss for each target mode. This suppression develops the selected multimode capability of the cavity through the introduction of slots.

The device incorporates a semiconductor cavity of length, \( L = 400 \mu m \) with \( n = 3.20 \) and the free space wavelength is selected at \( \lambda_0 = 1.6 \mu m \). Slots of length \( L_s = 0.126 \mu m \) are added to the device such that the quarter wavelength condition is satisfied in the cavity. Since the slot density function, Figure 2, concentrates them in groups along the length of the cavity, this limits the number of slots that may be introduced. Because of the slot’s length, they may be fabricated through electron-beam lithography, which will enable patterning on the same length scale as the optical wavelength in the cavity. The mirror loss spectrum of this cavity is shown in Figure 3, with spacing of 10 nm in the range of 1580 to 1620 nm. An average mirror loss reduction of 2.356 cm⁻¹ with respect to the cavity is seen for the chosen modes. The peak mode \( \lambda_0 = 1600 \mu m \) observes a 2.405 cm⁻¹ loss from the cavity and provides for the multimode selectivity of the device.

This device offers a simpler alternative for fabrication of a mode comb than the complex multi-wavelength DFB lasers incorporating binary superimposed gratings (BSG). This ultimately may permit for a tunable device utilizing coupled cavities for mode selectivity.

#### III.2. Device 2: The symmetric as-cleaved case

This device considers a semiconductor cavity of \( L = 2000 \mu m \) and \( n = 3.75 \). The facet reflectivities are designated as real and given by

\( r_1 = r_2 = (n-1)/(n+1) \). The free space wavelength is chosen at \( \lambda_0 = 75 \mu m \) with a corresponding mode, \( m_0 = 200 \). Next, 66 slots of length \( L_s = 5.027 \mu m \), with \( \Delta n = -0.02 \) are introduced into the cavity. Here, the cavity mode spacing is chosen as \( \alpha = 8 \) and \( \tau = 0.0895 \) such that there is an 80% suppression of the mirror loss modulation amplitude. As derived, the change in the threshold gain of the \( n^m \) cavity resonance, \( \Delta \gamma_{m} \), is proportional to the modulation amplitude function, \( f(\alpha) \) where

\[
f(\alpha) \sim \{ | r_1 \exp(-\alpha, L \alpha) - | r_2 | \exp(-\alpha, L \alpha) \}
\]  

(III.2)
\[ \alpha = \frac{1}{L} \ln \left( \frac{1}{r_1 r_2} \right) \]  

provides the mirror losses of the plain cavity. The behavior of this function with respect to the facet reflectivities forms the foundation of this research. The additional reflections caused by the presence of the slots in the cavity modulate the relatively uniform threshold gain of the FP cavity modes. The strength of this feedback is proportional to the position of the slots with respect to the facets. The symmetric case provides a trivial example of slot introduction into the cavity but the index pattern that emerges can be related to a phase-shifted DFB laser with a half wavelength subcavity at the center of the device and all others subject to quarter wavelength.

For the symmetric as-cleaved case, \( \varphi_1 = \varphi_2 = 0 \) and the modulation amplitude function, \( f(\epsilon) \sim \sinh \epsilon L a \), is odd, requiring a \( \pi/2 \) phase shift at the center of the device. Approximate slot positions \( \{ \epsilon_i \} \) are determined by evaluating an expression of the form given by Equation II.6 with Mathematica®. It should be noted that because the \( n = 0 \) term provides effectively a d.c. component here for the symmetric case, it is neglected. Also, the presence of a divergence in the slot density function, \( \left[f(\epsilon)\right]^{-1} \) at \( \epsilon = 0 \) requires \( \epsilon_{\text{min}} = \frac{L_s}{2} \), where \( L_s \) is slot length with respect to the cavity for integration. The implications of this divergence in the slot density function will be further examined in the anti-reflection (AR) and high-reflection (HR) coating case that follows. A schematic picture of this device is shown in Figure 4.

To generate the slot positions seen in Figure 4 a series of programs were written in FORTRAN, which take the list of \( \epsilon \), generated from Equation II.6 and correct for the optical path lengths due to the introduction of the slots and still meet the phase requirement. The formulation for the correction has been outlined above. Each line in Figure 4 represents the center of a slot (a perturbation in the uniform effective refractive index of the cavity); almost the entire device is filled, greatly increasing the spectral purity. The

\[ \text{Figure 2. Object slot density function (upper) and laser cavity schematic (lower) for multimode laser.} \]

\[ \text{Figure 3. Mirror losses for the 5-mode comb cavity with 14 index modulations. The horizontal line is at the value of mirror losses of the plain FP cavity.} \]
resulting mirror loss spectrum of the cavity is plotted in Figure 5.

As shown, introduction of a maximum number of slots, whereby $\kappa L < 1$ is maintained, provides for an index patterned loss of 0.811 cm⁻¹ at the selected mode. Here, $\kappa$ is defined as an average coupling coefficient and $L$ is the cavity length. This condition maintains the FP nature of the cavity. It can be shown that $\delta_m < \pi$, where $\delta_m$ represents the change in the cavity resonance condition due to the introduction of slots along the device as previously described. The reduction in mirror losses suggests an alternative laser source to the index-coupled DFB lasers developed by Mahler, et al, which achieved minor suppression while the regularly spaced FP spectrum was still observed. While the complex-coupled DFB will intrinsically operate at a higher threshold current due to the introduction of lossy defects in the laser cavity, this IPFP may provide for higher operating temperatures than those offered by Mahler, et al, with a decreased threshold current and increased slope efficiency.⁴

### III.3. Device 3: The AR–HR asymmetric case

This terahertz laser incorporates an anti-reflection (AR) and a high-reflection (HR) coating at the left and right facets ($r_l = 0.20$, $r_r = 0.95$) respectively, since, in practice, laser emission from one edge is desired for optimal output power. The modulation amplitude function $f(\varepsilon)$ has a zero at $\varepsilon = 0.469$ (which is designated as $\xi$) and therefore requires a $\pi/2$ phase shift at that point. The need for this phase shift at $\xi$ is required by the facet coatings of the device. Based on DFB grating theory, the $\pi/2$ phase shift at the center is used to remove the mode degeneracy in AR-coated index-coupled DFB lasers. Addition of the phase shift is more reliable method than using facet reflectivities but requires more complex fabrication. It should also be noted, that a DFB may incorporate just one AR facet or two. However, using the techniques of O’Brien and O’Rielly, coatings that yield no reflectivity on one or both facets will result in no laser emission.⁷ The IPFP laser presented provides for simpler fabrication while offering improved spectral purity.
The treatment used to find slot positions when a zero of the modulation amplitude function lies in the domain of the device is now presented. The slot positions to the left of the device center, \(-1/2 < \xi < 0\), are generated similarly to those in the symmetric case; as is expected, the slot pattern resembles the reconstruction of the Fourier spectrum and fills the entire cavity. The right section of the device requires an extension of Equation II.6 to compensate for the divergence in \([f(x)]^{-1}\). Evaluation of two integrals over the domains \(0 < \xi < \xi - \xi_j\) and \(\xi + \xi_j < \xi < 1/2\) given by

\[
\sum_{n=0}^{N-2} \int_{\xi_n}^{\xi_{n+1}} f(x)^{-1} \Gamma \left( x - \frac{n}{a} \right) dx,
\]

yield the necessary normalization coefficients used in Equation II.6. A finite number of slots are then selected to fill the entire right cavity. These slots are distributed with the normalization coefficients such that they lie before or after \(\xi\). Again, two evaluations of Equation II.6 are necessary to give the correct slot positions where \(\xi_{n=0} = 0\) and \(\xi + \xi_j/2\) respectively.

As seen in Figure 6, the slot density function diverges at \(\xi = 0.469\) and consequently, concentrates slots near that value; while the left side of the device appears to be regularly distributed. This behavior, in contrast to uniform gratings, supports the rationale for treating the left and right sides of the device independently so that a maximum number of slots can be introduced providing the improved spectral purity. Similar to DEVICE 1, the slot position requirement here, near \(\xi\), limits the number of slots that can be introduced. Although this results in a poor sampling of the object spectrum on the right of the device, an approximate 2 cm\(^{-1}\) mirror loss reduction is resolved for the principle wavelength. This work suggests that patterning the refractive index cavity may be a better approach for achieving spectral pure AR-HR THz devices than utilizing a continuous grating for an index-coupled DFB laser as developed by Mahler, et al, for devices where \(KL \sim 1\).

Comparatively, the complex-coupled DFB fabricated by Mahler, et al, achieved single-mode operation with an increase in the

Figure 6. Object slot density function (upper) and laser cavity schematic (lower) for AR/HR laser.

Figure 7. Mirror losses for the AR/HR laser cavity with 33 slots. The horizontal line is at the value of mirror losses of the plain FP cavity.
average waveguide losses of 4 cm⁻¹, at the expense of further processing and increased threshold current. The mirror loss spectrum for this IPFP cavity is plotted in Figure 7 and demonstrates the mode suppression (about 80%) offered by the introduction of 33 index steps. The slots offer a 1.907 cm⁻¹ loss at the targeted mode, while reducing the threshold current and increasing the slope efficiency.

IV. CONCLUSION

This research has shown how incorporation of a patterned index into a Fabry-Pérot cavity provides for improved spectral purity. It also demonstrates that the techniques conceived by O’Brien and O’Reilly may be extended to develop multimode cavities and allow for the design of spectrally pure lasers at various wavelengths. These Index Patterned Fabry-Pérot (IPFP) devices offer an alternative to DFB lasers for single-mode applications at reduced fabrication costs. Furthermore, at longer wavelengths, it is simpler to implement such index steps more accurately during the lithographic and etching stages of fabrication since the length of the perturbation is on the order of several microns. The IPFP alternative approach has been suggested for index-coupled DFB lasers that use a continuous grating. Future work aims to couple these IPFP cavities to create a tunable laser source. At present, an analytical expression of the complex transmission coefficient of a coupled cavity laser with these IPFP cavities has been found, and further work would develop the tunability of the device through carrier injection. Additionally, numerical methods to determine the validity of the first order approximation may be explored whereby multiple scattering would be considered for quantitative comparison to DFB lasers.

Acknowledgements

I would like to thank Dr. S. O’Brien (Tyndall Institute, formerly the NMRD) for his insight and guidance throughout this project as well as Professor E. O’Reilly (Tyndall Institute) for his support. This research was supported by the National Science Foundation (NSF) and Science Foundation Ireland (SFI).

REFERENCES

Transverse Vibrating Modes of Resonance Wire Loop

Aysha Chowdhry  
DEPARTMENT OF PHYSICS & ASTRONOMY

Elaine Tsao  
DEPARTMENT OF PHYSICS & ASTRONOMY

Arthur Schmidt  
CONTRIBUTOR & FACULTY MENTOR, DEPARTMENT OF PHYSICS & ASTRONOMY, NORTHWESTERN UNIVERSITY

ABSTRACT
The Resonance Wire Loop is a common demonstration apparatus often used in the physics class to visually represent standing waves of electrons in orbitals of the Bohr atomic model. The apparatus consists of a wire loop supported at a single point which also serves as the point of excitation. The resonance modes are easily demonstrable and patterns of resonance frequencies have only been explored recently. The harmonic frequencies of a wire hoop are unusual in that they are proportional to the square of the odd integers 1, 3, 5... etc. Furthermore, the fundamental frequency of the standing wave is proportional to the inverse of the squared circumference of the hoop. The observed resonance frequency structure of the wire formed into a circular hoop is more closely related to the transverse vibration modes of a straight stiff bar than to the vibration of a straight wire or string.

I. INTRODUCTION
The Resonance Wire Loop apparatus is a standard accessory to the Mechanical Wave Driver sold by Pasco Scientific. It involves a wire loop supported at one point which is also attached to a vibrating exciter as shown in Figure 1a. As one sweeps through the frequencies of vibration of the support point, transverse standing waves are excited in the wire loop at specific resonant frequencies and certain modes of oscillation are observed. In the fundamental mode, the entire loop oscillates in unison with the amplitude increasing with increased distance from the support reaching a maximum or single antinode at a point exactly opposite the support point. Higher frequency modes can also be excited with increased numbers
of antinodes. The next highest resonance occurs with three antinodes in a pattern symmetric about a diameter which passes through the support point. Figure 1 shows the apparatus with the wire oscillating in the mode having three antinodes. All resonance modes of oscillation have an odd number of antinodes and the patterns are symmetric about the diameter passing through the support point.

One of the more interesting aspects of this phenomenon is the pattern of resonance frequencies excited in the loop. A linear analog to this apparatus is the standing waves of a string fixed at both ends and excited at a point near one end. This standard physics demonstration exhibits standing waves whose frequencies are simple multiples of the fundamental frequency, which is determined by the length of the string and the tension in the string.

We have determined the pattern of frequencies of harmonic modes for the wire loop supported at a single point to be multiples of the square of odd integers, given by

$$f_n = f_1(n^2) \quad n=1, 3, 5...$$  \hspace{1cm} (1)

where \(f_n\) represents the frequency of the \(n^{th}\) harmonic mode. This differs from what we see in the case of standing waves on a string and has interesting implications about the waves being generated in this apparatus that are worth exploring.

Recently Danning Bloom and D. W. Bloom reported seeing a similar pattern of frequencies when they positioned the loop vertically and excited the oscillation in the plane of the loop, which is the way it is illustrated in the PASCO catalog. We, on the other hand, excited the loop transverse to plane of the loop with the loop suspended horizontally. This orientation produces standing waves with clearly observable nodes at specific points around the loop. An analytical solution to this arrangement may be more easily derived since the vibration motion is completely orthogonal to the direction of wave propagation and the plane into which the wire turns to make the loop. However, such a solution is beyond the scope of this present work.

The fundamental resonance frequency in the case of a standing wave on a string is determined by the speed on the string and the wavelength of the wave. The wavelength is determined by the mode of the resonance, the boundary conditions at the ends of the string and the length of the string. The speed of the wave, \(v\), depends on the tension, \(T\), and the density, \(\mu\), of the string, using the equation

$$v = \sqrt{\frac{T}{\mu}}$$  \hspace{1cm} (2)

where \(T\) is the tension in the string and \(\mu\) is the mass per unit length.

The boundary conditions and length of the string determine the size of the wavelength of the resonance. For example, a string fixed at both ends will have wavelengths given by

$$\lambda_n = \frac{nv}{2L}$$  \hspace{1cm} (3)

where \(n\) is the integer of the mode and \(L\) is the length of the string.

In the case of the wire loop, similar conditions hold. The wavelength of the resonance is determined by boundary conditions and the length of the wire, which, in the case examined, is the circumference of the loop. Since the support point is the source of waves that radiate in both directions around the loop, resonances must be symmetric with respect to the point of support, and only patterns with odd numbers of nodes can be excited. The wavelengths of these resonance waves are simple odd fractions of the circumference. In terms of the diameter, \(d\), of the loop, the relationship is given as

$$\lambda_n = \frac{2\pi d}{n} \quad \text{for} \quad n=1, 3, 5...$$  \hspace{1cm} (4)
There are several details to be mentioned in this set-up. In the linear analog the point of excitation of the vibrations where the mechanical vibration is introduced into the system is near the end of the string at the point of attachment of the string. In the loop, there is only the one support point which is also the point where the periodic excitation is introduced to the system. Thus, it cannot be a node and a point of excitation. As the source of the disturbance, the support point radiates a wave in both directions away from the support. This results in resonances that are always symmetric with respect to the line bisecting the circle. In fact, two nodes are formed to either side of the point of support. Figure 1 shows a close up of the support for the harmonic mode with three antinodes. Technically, the post forms a fourth antinode which is small compared to the other antinodes.

II. EXPERIMENT
The Resonance Wire Loop apparatus comes from Pasco with a single wire loop of circumference 80 cm and thickness 1 mm. We expanded on this setup by obtaining four different thicknesses of steel piano wire. For each wire thickness, pieces of varying length were cut and formed into loops of different diameters. The wires were mounted on a post attached to the center of a speaker cone. A waveform generator powered the speaker. Each wire was mounted in the apparatus and as many resonance frequencies were measured as could be excited.

III. RESULTS
The resonance patterns were examined over a range of frequencies from the fundamental through to the highest frequency for which a resonance mode could be observed, and the positions of nodes and antinodes were noted. The relationship expressed in Equation 4 was experimentally verified from these observations. For example, one can see in Figure 1, where the hoop is oscillating in the \( n = 3 \) mode, there are three antinodes corresponding to 1.5 wavelengths around the circumference of the hoop. In this case the wavelength is \( 2/3 \) of the circumference or \( \lambda_s = 2\pi d/3 \).

To verify the frequency formula (Equation 1) we plotted the resonance frequencies as a function of \( n^2 \) (square of the mode number). The plot appears in Figure 2. To obtain an accurate fundamental frequency, we took each frequency in the sequence of harmonics for a given geometry and divided it by the squared integer of the mode. We then averaged these values and calculated a standard deviation. We obtained values of the fundamental frequency for

![Figure 2](image-url)
each loop circumference using wires of the same thickness. Figure 3 shows the relationship between the fundamental frequency obtained this way and the loop circumference. A nearly perfect inverse quadratic relationship is shown. We then repeated the measurements over the four thicknesses of wire with similar results.

If the wire forming the loop were opened up and straightened into a line, the fundamental frequency would be inversely related to the length of the wire. In the loop arrangement the fundamental frequency seems to be related to the inverse square of the wire length as is circumference. This relationship was observed over the range of wire thicknesses tested.

IV. CONCLUSION

In the case of a standing wave on a string the resonance frequency arises from the condition of wavelengths allowed by the boundary conditions and the speed of the wave on the string. The speed is independent of the frequency of the wave at least to first order. Similarly, from observing a pattern of nodes and antinodes we determined the wavelength of the transverse wave on the loop while noting the frequencies that correspond to the resonance modes. With this information we have deduced the speed of the wave excited on the loop. From the unique quadratic relation of frequency to resonance order \( n \) as expressed in Equation 1 we may consider the possibility that the speed of the wave is not constant over the various resonance modes. It may instead be somehow strongly dependent on the frequency of the exciting oscillation. If the basic relationship between frequency \( f \), wavelength \( \lambda \) and wave speed \( v \) is to hold for resonance mode \( n \)

\[
f_n \lambda_n = v_n
\]

then the velocity must be strongly frequency-dependent, as expressed by the equation

\[
v_n = k \sqrt{f_n}
\]

where \( k \) is a constant which depends on geometric and material characteristics of the wire. This is very similar to what is seen in a stiff straight bar.\(^2\) In that case both tension along the bar and the stiffness of the bar contribute to the restoring force that is responsible for the velocity of the wave. We suspect that additional tension may be introduced in the case of the circular wire by bending the wire into a loop. Additional tension may also be introduced as a result of the increased rate of change of displacement of the higher frequency modes.

Since the loop consists of a stiff wire we might consider a comparison to the modes of transverse vibrations for a stiff wire. Rossing,\(^1\) in his book *The Science of Sound*, gives the formula for the frequencies of the harmonic series for a piano string as

\[
f_n = nf_1[1 + (n^2 - 1)A] \quad \text{for} \quad n = 1, 2, 3\ldots
\]

where \( f_n \) is the frequency of the \( n^{th} \) harmonic.
and $f_i$ is the frequency of the fundamental. For a solid wire

$$A = \frac{\pi^3 r^4 E}{8TL^3} \tag{8}$$

where $r$ is the radius of the wire, $E$ is Young's Modulus, $T$ is the tension, and $L$ is the length of the wire.

The observed proportional relationship between fundamental frequency $f_i$ and the inverse square of the circumference is also similar to the relationship between frequency and length of a stiff straight bar.

Rossing also notes that for vibrations in a stiff bar the frequency depends on $L^2$ rather than $L$, as in a vibrating string and the modes of frequency go as the square of odd integers. The modes sequence begins with $M = 3.0112$ and then continues with integer values 5, 7, 9... This does not correspond exactly with our results in which the mode sequences begin with $n = 1$. Measurements of greater precision might shed more light on the analysis.

Future work might involve actual measurements of wave speeds for transverse wave trains traveling around the hoop. Such direct observations of wave velocities would require ultra high speed imaging that might challenge the present technology.

Acknowledgments
The authors are very grateful to Prof. Thomas Rossing for his valuable assistance in preparing the paper.

REFERENCES

Decoupled Control for the Snakeboard

Benjamin Jay Stephens
DEPARTMENT OF MECHANICAL ENGINEERING

Kevin M. Lynch
FACULTY ADVISOR, DEPARTMENT OF MECHANICAL ENGINEERING, NORTHWESTERN UNIVERSITY

ABSTRACT

The snakeboard is a good example of a complex dynamic system. It is constrained by the nonholonomic properties of its wheels, and under-actuated in that there are more degrees of freedom than control inputs. This paper presents a simplified approach to the derivation of the reduced equations of motion. Using these equations to derive decoupled non-linear controls allows a robotic snakeboard to follow a path under feedback control. The controllers represent a decoupling of the kinematics and dynamics of the snakeboard body. Similarities between the theoretical and experimental results, which are both presented, demonstrate the validity of our model.

I. INTRODUCTION

The snakeboard is a commercial toy that resembles a skateboard. On a snakeboard, however, the rider can change the angle that the wheels make with the body. By swaying the arms and body, the rider can propel the snakeboard through conservation of momentum, much like a person experiences in a swivel chair. Here, however, the wheel angles change the center of rotation, allowing a net motion. Without ever touching the ground and starting at rest, the rider of the snakeboard can move anywhere on a surface. A robotic snakeboard works in a similar fashion. Without ever directly powering its wheels, it too can move anywhere on a surface.

Because of unmodeled dynamics, such as friction, full-state trajectory tracking is impossible for the snakeboard; energy will
inevitably be wasted by some means in a real-world system. This paper shows that a natural decoupling of input forces makes path-tracking a simple task. If position on a path is more critical than velocity, it is simpler to divide the controls into steering and energy-generating. Then we can use steering inputs to steer the body onto a reference path and use inputs that affect the energy of the system to control the kinetic energy or speed.

This paper describes and derives decoupled non-linear controls for the snakeboard that allow a robotic snakeboard to follow a predefined path while maintaining a constant kinetic energy. It is divided into three sections: developing the mathematical model for the snakeboard, deriving the decoupled controls, and describing the experimental setup. Both simulated and experimental results are included.

This is not the first paper to investigate the snakeboard. Since it is such a complex system, there has been a wide variety of approaches to this problem. The problem was first introduced in a 1994 paper, which demonstrated the kinematic controllability and investigated the use of gait motions and sinusoidal variations of the control inputs for the snakeboard. The snakeboard system has also been described using geometric mechanics. Kinematic motion planning as well as optimal control have been investigated. The existence of control switches, which involves a finite minimum number of motions required to reach a destination point, for use in motion planning has also been researched. The most recent work has derived control for the snakeboard using averaging theory, on-the-fly adjustments to the control inputs to compensate for position error. To our knowledge, the type of control for the snakeboard presented in this paper has not been previously presented.

II. THEORETICAL MODEL

This section includes the derivation of the kinematics and reduced dynamics for our snakeboard model. Our snakeboard is a simplified model of the real snakeboard with its steering wheels constrained to turn symmetrically, as seen in Figure 2.

II.1 Kinematics

The kinematics of our snakeboard model are simplified because the front and rear steering wheel angles are the same. In fact, it has been shown that the kinematics of such a snakeboard reduce to the kinematics of a "car-like" robot, which has an \((r, y, \theta)\)-position in the 2D \(xy\)-plane. The two other coordinates of the snakeboard are the steering angle, \(\phi\), and the flywheel angle, \(\psi\). The configuration, \(q\), of the snakeboard is

\[
q = \begin{pmatrix}
x \\
y \\
\theta \\
\psi \\
\phi 
\end{pmatrix}
\]  

(1)

The snakeboard is subject to nonholonomic velocity constraints, caused by an assumed "no-slip"
condition on the wheels. These constraints have the form, $A(q) \dot{q} = 0$, where the Pfaffian constraint matrix,

$$A(q) = \begin{bmatrix}
\sin(\theta + \phi) & \cos(\theta + \phi) & -l \cos \phi & 0 & 0 \\
\sin \theta & \cos \theta & 0 & 0 & 0
\end{bmatrix}$$

(2)

is identical to that of a "car-like" robot. These constraints cause the snakeboard to rotate about an instantaneous center of rotation that lies on a line perpendicular to the body. The corresponding radius is determined by the angle of the steering wheels and the distance, $l$, as shown in Figure 2.

$$r = l \cot \phi$$

(3)

Given an angular velocity of the body, $\dot{\theta}$, the linear velocity,

$$v = \dot{\theta} r = \dot{\theta} l \cot \phi .$$

(4)

The kinematics of the snakeboard can be written as

$$\dot{q} = \begin{bmatrix}
\frac{l \cos \theta \cot \phi}{l \sin \theta \cot \phi} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\dot{\theta} \\
\dot{\psi} \\
\dot{\phi}
\end{bmatrix} .$$

(5)

11.2. Dynamics

The dynamics of the snakeboard involve rigid body motion in a plane. In this section, the reduced dynamic equations of motion will be derived using a Lagrangian mechanics approach.

11.2.1. Lagrange's Equations of Motion

The snakeboard is a dynamic system with nonholonomic constraints. Therefore, we use Lagrange's equations to find the equations of motion. Lagrange's equations of motion for a constrained system are

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau + A^T \lambda ,$$

(6)

$$A \dot{q} = 0$$

where $\tau$ is a vector of generalized control torques and $A(q)$ is the Pfaffian constraint matrix, given for the snakeboard in Equation 2. $L(q, \dot{q})$ is a Lagrangian equal to

$$L(q, \dot{q}) = \dot{q}^T M(q) \dot{q} ,$$

where $M(q)$ is the mass matrix of the system. The Lagrangian is a scalar quantity that represents the kinetic energy of the entire system. The mass matrix for the snakeboard is

$$M(q) = \begin{bmatrix}
m & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 \\
0 & 0 & J_\theta & J_\psi & 0 \\
0 & 0 & J_\psi & J_\phi & 0 \\
0 & 0 & 0 & 0 & J_w
\end{bmatrix} ,$$

(8)

where $J_\theta$, $J_\psi$, and $J_\phi$ are the moment of inertia of the body, flywheel, and wheels, respectively, and $J_w = J_\theta + J_\psi + J_\phi$. The Lagrangian for the snakeboard is

$$L(q, \dot{q}) = \frac{1}{2} m(x^2 + y^2) + \frac{1}{2} J_\theta \dot{\theta}^2$$

$$+ \frac{1}{2} J_\psi \dot{\psi}^2 + \frac{1}{2} J_\phi \dot{\phi}^2 .$$

(9)

Note that neither the Lagrangian nor the mass matrix depend on the current configuration of the snakeboard. Since $q$ does not appear in the Lagrangian for the snakeboard, $\partial L / \partial q = 0$. By inserting Equation 9 into Equation 6, the dynamics simplify to

$$m \ddot{x} = \lambda_x \sin(\theta + \phi) + \lambda_\psi \cos(\theta + \phi)$$

$$m \ddot{y} = -\lambda_x \sin(\theta + \phi) + \lambda_\psi \cos(\theta + \phi)$$

$$J_\theta \ddot{\theta} + J_\psi \ddot{\psi} = \tau_\theta + \tau_\psi$$

$$J_w \ddot{\phi} = \tau_\phi ,$$

(10)

where $\tau_\theta$ and $\tau_\psi$ are the input torques to the flywheel and steering wheels, respectively.

11.2.2. Reduced Equations of Motion

Lagrange's equation, Equation 6, for a mechanical system can also be rewritten as

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} = A'(q) \lambda + T(q) \tau ,$$

(11)

where $C(q, \dot{q})$ contains the Coriolis and Centrifugal terms. However, for the snakeboard, $C(q, \dot{q}) = 0$ because there is no dependence on state in the inertia matrix. If we first multiply both sides of Equation 11 by a matrix, $D^T$, such that $D^T A'(q) \lambda = 0$, we can eliminate $\lambda, x$, and $y$ from Equation 10. The result is

$$D^T M(q) \ddot{q} = D^T T(q) \tau .$$

(12)
The $D^T$ for the snakeboard can be found from the kinematics, and is equal to
\[ D = \begin{bmatrix} l \cos \theta \cot \phi & 0 & 0 \\ l \sin \theta \cot \phi & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \] (13)

If $q = (\theta, \psi, \phi)$, then $\dot{q} = D \dot{q}$, and
\[ \ddot{q} = D \ddot{q} + \hat{D} \dot{q}. \] (14)

If we substitute this into Equation 12, we get
\[ D^T M(q) D \ddot{q} + D^T M(q) \hat{D} \dot{q} = D^T T(q) \tau, \]
which can be written as
\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} = \tau, \] (15)
where $\tau = D^T T(q) \tau = (\tau_\theta, \tau_\psi, \tau_\phi)^T$. We can compute the reduced inertia matrix, $M(q)$, by
\[ M(q) = D^T M(q) D = \begin{bmatrix} ml^2 \cot^2 \phi + J_\theta & J_\psi & 0 \\ J_\theta & J_\phi & 0 \\ 0 & 0 & J_w \end{bmatrix}. \] (16)

Likewise, we can calculate the reduced $C(q, \dot{q}) \dot{q}$ matrix (which is not zero) by
\[ C(q, \dot{q}) \dot{q} = \begin{bmatrix} -ml^2 \phi \csc^2 \phi \cot \phi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \] (17)

Using Equations 15, 16 and 17, we can write the reduced equations of motion as
\[ (ml^2 \cot^2 \phi + J_\theta) \ddot{\theta} + J_\psi \ddot{\psi} - ml^2 \phi \csc^2 \phi \cot \phi \dot{\phi} = 0 \] (18)
\[ J_\phi (\ddot{\theta} + \dot{\psi}) = \tau_\theta \] (19)
\[ J_w \ddot{\phi} = \tau_\phi \] (20)

II.2.3. Power in the Snakeboard

Since the Lagrangian is equal to the kinetic energy, $KE$, a simplified equation for the kinetic energy using the reduced inertia matrix from Equation 16 can be written as
\[ KE_{\text{sys}} = \frac{1}{2} \left[ (ml^2 \cot^2 \phi + J_\theta) \dot{\theta}^2 + J_\phi (\dot{\theta}^2 + \dot{\psi}^2) + J_w (\dot{\phi}^2 + \dot{\psi}^2) \right]. \] (21)

For just the body of the snakeboard, the kinetic energy can be written as
\[ KE_{\text{body}} = \frac{1}{2} (ml^2 \cot^2 \phi + J_\theta) \dot{\theta}^2. \] (22)

The power is equal to the time derivative of the kinetic energy. By taking the derivative of Equation 21 and Equation 22 and substituting in the dynamics equations, the powers of the snakeboard system and snakeboard body become
\[ P_{\text{sys}} = \tau_\theta \dot{\psi} + \tau_\phi \dot{\phi}, \] (23)
and
\[ P_{\text{body}} = -J_\phi \dot{\phi} \dot{\phi}, \] (24)
respectively. Equation 23 has an expected form (power = torque × angular velocity). The motors act like an internal power source and are used to change the level of kinetic energy in the system.

III. CONTROL

The control strategy presented in this paper decouples the kinematic and dynamic control of the snakeboard. A kinematic path-tracking controller treats the snakeboard as a car-like system with the goal of converging to a predefined path in Cartesian space. Because the snakeboard cannot converge to a path under zero velocity, a dynamic energy controller adds kinetic energy to the snakeboard body, effectively controlling velocity. In both cases, the controllers are assumed to have full access to all of the state variables.

External motion planners will be assumed to provide the reference paths that these controls use. For the snakeboard, this means that a path in $(x, y, \theta)$-space is provided which satisfies the nonholonomic constraints of the wheels. Since the controller will be implemented on a real-world system, a feedback controller is used. This allows for recovery from small perturbations in the modeled dynamics and for more robust path-tracking.

III.1. Kinematic Control

The goal for kinematic control of the snakeboard is to track a path in $(x, y, \theta)$-space. By ignoring the dynamics and using a reduced set of coordinates, the snakeboard becomes a kinematic
system, much like a car with two velocity inputs. Feedback controllability for this type of system has been widely studied.\textsuperscript{11,12,13}

The snakeboard is assumed to follow the kinematics given by Equation 5. The feedback controller presented in this section is similar to the previously conceived controller,\textsuperscript{14} which minimizes a position error function by specifying a desired angular velocity, $\dot{\theta}_d$, of the snakeboard body. The driving velocity, $v$, is assumed to be given, as it is achieved by the dynamic controller. Equation 4 relates $\dot{\theta}$ to the steering angle, $\phi$. Given the output, $\dot{\theta}_d$, from the feedback controller and the current velocity, the desired steering angle is

$$\phi_d = \tan^{-1}\left(\frac{\dot{\theta}_d}{v}\right).$$

If the control is chosen to be $u = \tau$, then the desired steering angle in Equation 25 can be achieved by a PID control,

$$\tau_e = k_\tau \dot{e}_\theta + k_d \dot{e}_\theta + \int_0^s e_\theta ds,$$

where $e_\theta = \phi_d - \phi$, and the gains, $k_\tau$, $k_d$ and $k_\theta$, are tuned accordingly.

### III.1.1. Reference Configurations

The reference configuration is denoted by $q_r = (x_r, y_r, \theta)^T$, and is located on a reference path, $R(s) = (x_r(s), y_r(s), \theta_r(s))^T$, parameterized by a path parameter, $s$. If the snakeboard has a current configuration, $q_c = (x_c, y_c, \theta_c)^T$, then the reference configuration is the point, $R(s_{\text{min}})$, where

$$s_{\text{min}} = \arg \min_{s \in (0, \pi)} \| q_c - R(s) \|,$$

where the norm, $\| q_c - R(s) \|$, is defined by

$$\| q_c - R(s) \| = \sqrt{(x_c - x_r(s))^2 + (y_c - y_r(s))^2 + d^2(\theta_c - \theta_r(s))^2},$$

Here, $d$ is the radius of gyration of the snakeboard. Therefore, $q_c$ is the point on $R(s)$ such that the distance between $q_c$ and $q_r$ is a minimum.

### III.1.2. The Error Configuration

The error function that is to be minimized by the kinematic controller, $q_e$, is defined as the error configuration. It is found by the transformation,

$$q_e = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c & 0 \\ -\sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} (q_c - q_r).$$

As can be seen in Equation 28, if $q_e = 0$, then $q_c = q_r$. Figure 3 shows the geometric interpretation of this configuration transformation.

### III.1.3. Kinematic Control Algorithm

This section describes the controller selected for path tracking and shows that it is locally stable around the error configuration, $q_e = (0, 0, 0)^T$. In order for the error configuration to converge to zero, $\dot{q}_e$ should drive the error to zero. From Equation 28,

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{pmatrix} = \begin{pmatrix} \dot{\theta}_d x_e + v_e \cos \theta_c - 1 \\ -\dot{\theta}_d y_e + v_e \sin \theta_c \\ \dot{\theta}_d - \dot{\theta}_d \end{pmatrix},$$

where $\dot{\theta}_d$ is given by the control law, and $v_e$ and $\dot{\theta}_d$ are reference velocities given by

$$\begin{pmatrix} \dot{\theta}_d \\ v_e \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}_e}{\dot{s}} \\ \frac{\dot{y}_e}{\dot{s}} \end{pmatrix} = \begin{pmatrix} \sqrt{\left(\frac{\dot{x}_e}{\dot{s}}\right)^2 + \left(\frac{\dot{y}_e}{\dot{s}}\right)^2} \\ \frac{\dot{s}}{\dot{s}} \end{pmatrix}$$

where $s$ represents the speed of the snakeboard along a path. The control law for the snakeboard is

$$\dot{\theta}_d = \dot{\theta}_d + K_v v_{\text{ref}} + K_{\theta} v_e \sin \theta_c.$$
By inserting Equation 31 into Equation 29 and linearizing about the point, \( q = (0, 0, 0) \),

\[
\dot{q} = Aq,
\]

where

\[
A = \begin{bmatrix}
0 & \dot{\theta}_j & 0 \\
-\dot{\theta}_j & 0 & v_j \\
0 & -K_y v_j & -K_y |v_j|
\end{bmatrix}
\]  

(32)

The coefficients of \( s \) in the characteristic equation, \( \det(sI - A) \), are

\[
a_1 = 1 \\
a_2 = K_y |v_j| \\
a_3 = K_y v_j^2 + \dot{\theta}_j^2 \\
a_4 = K_y |v_j| \dot{\theta}_j^2
\]

These coefficients are always positive. Also,

\[
a_1 a_2 - a_3 a_4 = K_y K_y |v_j|^2 |v_j| \geq 0
\]  

(33)

By Equations 34 and 35 and the Routh-Hurwitz Criterion, the controller in Equation 31 is locally stable around the error configuration, \( q = (0, 0, 0) \).

**III.2. Dynamic Control**

The controller presented in this section is designed to keep the skateboard body at a desired level of kinetic energy, described by Equation 22. We will assume that the desired kinetic energy, \( KE_d \), will be a constant, or step, input. The control, \( u_j \), applies a force to the skateboard body that either adds or subtracts kinetic energy.

Below is the derivation of the controller that is shown to decrease a Lyapunov function, \( V \), where \( V \geq 0 \). The proposed Lyapunov function is

\[
V = \frac{1}{2} \varepsilon^2
\]

(36)

where \( \varepsilon = KE_j - KE_{\text{des}} \). To be a stable controller, the Lyapunov function must always be decreasing, \( \dot{V} \leq 0 \). This is simply

\[
\dot{V} = \varepsilon \dot{\varepsilon}
\]

(37)

From Equation 24, we get

\[
\dot{\varepsilon} = J_y \dot{\theta} \dot{\psi}
\]

(38)

If we make the control equal to the flywheel torque, \( u_j = \tau_y \), we get

\[
\dot{\theta} = \left( \frac{1}{J_y} u_j - \dot{\psi} \right)
\]

(39)

If this equation is substituted into the dynamics equation, Equation 18, and solved for \( \dot{\psi} \), then Equation 38 reduces to

\[
\dot{\psi} = \dot{\psi} \left( \frac{\alpha(\phi) u_j - b(\phi) \dot{\theta}}{\alpha(\phi) - J_y} \right)
\]

(40)

where

\[
\alpha(\phi) = ml^2 \cot^2 \phi + J_y
\]

and

\[
b(\phi) = J_y ml^2 \csc^2 \phi \cot \phi
\]

(41)

Inserting Equation 39 into Equation 37 gives \( \dot{V} \) in terms of the control,

\[
\dot{V} = \frac{\varepsilon \dot{\theta}}{\alpha(\phi) - J_y} \left( \alpha(\phi) u_j - b(\phi) \dot{\theta} \right)
\]

(42)

In order for Equation 42 to be less than zero, the control should be

\[
u_j = -K_y \varepsilon \text{sgn} \dot{\theta} + \frac{b(\phi)}{\alpha(\phi)} \dot{\theta}
\]

(43)

When Equation 43 is substituted back into Equation 42, the derivative of the Lyapunov function becomes

\[
\dot{V} = \frac{K_y \varepsilon \alpha(\phi) \dot{\theta} \text{sgn} \dot{\theta}}{\alpha(\phi) - J_y} \leq 0
\]

(44)

By Lyapunov stability, the dynamic controller will accept a step input, allowing the skateboard to locomote with constant body kinetic energy.

**IV. EXPERIMENTAL RESULTS**

The mechanical skateboard developed here in the Laboratory for Intelligent Mechanical Systems, LIMS, tries to reproduce the motion of a person riding a skateboard. In place of a rider, the mechanical skateboard has two motors and a flywheel. One motor rotates the steering wheels and the other rotates the massive flywheel, which acts as the human torso. The skateboard is connected to a computer workstation through
a ServoToGo card. The workstation runs the QNX real-time operating system on a 200MHz Pentium™ processor.

The snakeboard control program runs in real-time at 2KHz. It accepts information from the encoders on the motors and training wheels and calculates its current position, velocity, and energy. Then the program outputs the control signals, given by the kinematic and dynamic control laws, to the motors. The reference path is coded into the program, with the ability to load externally generated paths. The nearest reference point is calculated through an iterative process, searching forward and backward along the path for the $R_s_{\text{min}}$. Data is recorded in real-time and saved to a binary file. This file is read, through FTP, and analyzed by a MATLAB program on a Windows workstation.

Simulations of these controls on the snakeboard are shown in Figure 5 and experimental results are shown in Figure 6. These figures show both the body kinetic energy and position of the snakeboard converging to a desired value. The kinematic controller is given a sinusoidal reference path and an initial configuration of the snakeboard is off the path. The dynamic controller is given a step input, controlling a constant kinetic energy level in the body.

V. CONCLUSION

The control approach presented in this paper allows the snakeboard to follow any planar path while maintaining a constant energy. Because the flywheel can only be accelerated to a finite maximum speed, the ability to add energy to the system can be limited by the curvature of the provided path. For example, it is very difficult for the snakeboard to follow a circle or a line for an extended period of time.

The control was tested through both simulation and in experiment, both yielding similar results. These similar results can be attributed to our model, with the small variances in performance between the simulated and experimental results being attributed to the presence of friction and torque/velocity limits in the mechanical system.
By decoupling the input forces into steering and energy control, we simplified the snowboard system to one that is simple to understand and control. Kinematically, the snowboard was treated as a car-like robot in the plane, with the dynamics providing a near-constant velocity, accomplished by controlling the kinetic energy of the body. This is an example of how simple concepts can be combined for use in a complicated application such as the snowboard.

REFERENCES


Investigations of Nanoscale Ferromagnetism in InAs/InMnAs Core/Shell Nanowires Using Magnetic Force Microscopy

Dinna G. Ramlan
DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING

Lincoln J. Lauhon
FACULTY ADVISOR, MATERIALS SCIENCE AND ENGINEERING, NORTHWESTERN UNIVERSITY

ABSTRACT

Magnetic force microscopy (MFM) was used to characterize the magnetic properties of InAs/InMnAs core/shell nanowires grown by the vapor-liquid-solid method. Magnetic field-dependent phase contrast was observed in MFM images and was correlated with small protrusions observed in topographic images. Transmission electron microscopy (TEM) studies of nanowire structure and composition identified hexagonal MnAs precipitates on the surface of InAs nanowires. Variable temperature MFM studies from 300-350 K showed a decrease in magnetic contrast with increasing temperature, consistent with the Curie temperature of 318 K for bulk hexagonal MnAs.

I. INTRODUCTION

Spintronic technology\(^1\) may lead to new kinds of digital electronics that can process and store data on the same device, resulting in computer processors that work faster and consume less electricity. One approach to semiconductor-based spintronics is to alloy small amounts of magnetic elements with nonmagnetic semiconductors to produce diluted magnetic semiconductors (DMSs).\(^1\) For instance, the addition of Mn as an acceptor to InAs makes InAs a ferromagnetic p-type semiconductor. The incorporation of DMS materials in practical applications requires that the DMS material have a ferromagnetic transition temperature (\(T_c\)) above room temperature. Because small device sizes are important to minimize power dissipation and maxi-
mize speed, it is also important that the magnetic properties of the DMS materials at small length scale are understood. Recently, ferromagnetic p-type (In,Mn)As thin films with a $T_c$ of $\sim$330 K have been grown by metalorganic vapor phase epitaxy (MOVPE).\textsuperscript{2,3,4} The magnetic properties of (In,Mn)As in bulk or thin films have been studied,\textsuperscript{5,6,7} but the properties of nanostructures are relatively unexplored. The possibility of synthesizing nanoscale semiconductors in the form of "nanowires" has motivated us to study the magnetic properties of (In,Mn)As in one-dimensional nanostructures. Magnetic force microscopy (MFM) is an attractive method to study the magnetic properties of nanostructures due to its sensitivity to small magnetic moments and high spatial resolution. MFM was successfully used to interpret the orientation of the magnetic moments of Co and Co-Cu multilayered nanowires.\textsuperscript{8,9} In this paper we present our investigations of the nanoscale ferromagnetism in InAs/(In,Mn)As core/shell nanowires using MFM.

II. EXPERIMENTAL PROCEDURE

InAs/(In,Mn)As core/shell nanowires were grown on GaAs(111) or Si(111) substrates by the atmospheric pressure metal-organic chemical vapor deposition (MOCVD) in collaboration with Steve May of the Wessels group (Figure 1). This technique utilizes arsine (AsH$_3$), trimethyl-Indium (TMIn) and tricarbonyl (methylcyclopentadienyl) manganese (TCMn) as the metal-organic precursors with hydrogen as the carrier gas. In this process, Au nanoparticles were placed on the substrate as the catalyst for one-dimensional nucleation of the nanowire via vapor-liquid-solid (VLS) growth mechanism (Figure 2).\textsuperscript{10,11} In VLS growth, metal nanoparticles are used to catalyze the decomposition of the reactant gas (Figure 2a). The decomposed reactant gas then forms a solid solution with the metal. The increasing concentration of the reactant in the metal causes the supersaturation of the solid solution; as a result, one dimensional growth takes place (Figure 2b).

In a previous study, the direct incorporation of Mn in InAs was pursued by flowing TCMn during InAs growth. However, the decomposition of

Figure 1. The SEM micrographs of (a) the InAs/(In,Mn)As core/shell nanowires grown perpendicular to the epitaxially on GaAs(111) substrate and (b) at higher magnification, the formation of precipitates on the surface of the nanowires can be observed.
Figure 2. The schematic representation of the synthesis of core/shell nanowire by VLS growth mechanism taken from Lauhon et al.\textsuperscript{12} (a) The metal nanoparticle (orange) catalyzes the one-dimensional nucleation and growth of the nanowire (blue). (b) Radial growth was avoided because the catalytic reactant decomposition is favored. (c) The change in reactant activates the radial growth to form the nanowire shell (green).
TCMn on the nanowire surface leads to secondary nanowire growth on the sides of the nanowires. Hence, we attempted to grow core/shell nanowire structure (Figure 2c) with InAs as the core and (In,Mn)As as the shell. Nanowire synthesis was initiated with the growth of InAs nanowires at 400°C for several minutes, followed by the introduction of TCMn and the reduction of TMIn in the system for radial shell growth of (In,Mn)As at a higher temperature. The growth conditions of the (In,Mn)As shell were based on the growth conditions of (In,Mn)As thin films that exhibit room temperature ferromagnetism. The nanowires need to be well separated on an appropriate substrate for characterization of a single nanowire. For this purpose, the nanowires on the growth substrate were suspended in ethyl or isopropyl alcohol after the growth (Figure 3a). Then, the nanowires in solution were placed in ultrasonic bath for several seconds to separate the nanowires from the substrate (Figure 3b). Next, droplets of nanowires in solution were pipetted on the SiO₂ substrate (Figure 3c), and the substrate was left for several minutes for the alcohol to evaporate. The sample was then observed under optical microscope to identify the presence of the nanowires. These steps are repeated accordingly until several single nanowires can be identified on the substrate. The SiO₂ substrate was patterned with Au "grids" that serve as the "map" of the substrate. The positions of the nanowires with respect to the Au grids were determined by looking at the sample using an optical microscopy and/or scanning electron microscope (SEM) (Figure 3d and 3e).

In this study, a Digital Instruments Scanning Probe Microscope (Nanoscope IIIa) was used to perform the MFM. The MFM was conducted using cobalt coated magnetic tip, with the resonance frequency of approximately 75 kHz. The contrast of the MFM image is measured from the phase shift of the cantilever arising from the

Figure 3. The schematic diagram of wire dispersal in liquid and deposition on marked SiO₂ substrate. (a) The nanowires on the growth substrate are suspended in ethyl or isopropyl alcohol. (b) The nanowires separate from the substrate after the sonication process. (c) After the drop of nanowires in solution is pipetted on the marked SiO₂ substrate, the alcohol evaporates leaving the separated nanowires on the substrate. The (d) SEM and (e) optical microscope image show the positions of a nanowire with respect to the Au grids on the SiO₂ substrate; the nanowires are circled in red and blue respectively.
force gradient between the sample and the tip. This force gradient is proportional to the magnetization of the sample. Magnetization parallel (and antiparallel) to the magnetization of the tip will cause dark (and bright) contrast. The magnetic domains of the nanowires can be observed as the contrast in the MFM image that is constructed from the phase shift of the cantilever.

Variable temperature MFM was conducted above room temperature using a variable temperature scanner to analyze the temperature dependence of the magnetism of the nanowire and to estimate the $T_c$ of the nanowire. The scanner utilizes an applied magnetic field of 500 G to hold the sample fixed on the stage. When conducting variable temperature MFM, the temperature of the substrate was determined using a thermocouple before and after each scan.

III. RESULTS

Figure 4a and 4b are the AFM and MFM images of the InAs/(In,Mn)As core/shell nanowires. The MFM image shows dark contrast due to magnetic domains that are parallel to the magnetization of the tip. It was found that the dark contrast corresponds to protrusions from the surface of the wire. This can be seen clearly by looking at the section analysis along the axis of the wire (insets of Figure 4); protrusions from the surface caused negative phase shift in the phase response of the cantilever.

The energy dispersive x-ray spectroscopy (EDS) of transmission electron microscopy (TEM) was used to determine the elemental composition of the nanowires (Figure 5). The TEM micrograph (insets of Figure 5) shows that there is a precipitate that sits on protrusions from the surface of the wire. The EDS spectrum of the precipitate (Figure 5a) indicates the presence of Mn and As. In peaks can also be identified but the peaks are of low intensities compared to those of Mn and As. This suggests that the protrusions observed in the MFM measurements are the ferromagnetic MnAs phase that appear as precipitates on the nanowire surface of the size of 5 nm to 30 nm in diameter. The EDS spectrum of the nanowire (Figure 5b) yields only In and
Figure 5. (a) The EDS spectrum of the precipitate taken at approximately the position of the red dot shows that the precipitate consists of mainly Mn and As with low concentration of In and (b) the EDS spectrum of the nanowires taken at approximately the position of the blue dot shows that the nanowires only consists of In and As; no Mn peaks were detected in this spectrum. The additional unlabeled peaks are attributed to the Cu membrane that was used to support the nanowires in the TEM sample.
As peaks with no indication of Mn peaks. In addition, there was no indication of a layer of (In,Mn)As on top of InAs nanowire. Because the precipitates are identified as MnAs, it is evident that phase segregation occurs during the incorporation of Mn in the (In,Mn)As shell growth with MnAs existing as the additional phase. TEM was also used to generate the diffraction pattern of the nanowires but the analysis will not be discussed in this paper. We found that both MnAs precipitate and InAs nanowires have a hexagonal crystal structure. The nanowire grows along the [0001] direction (or equivalently [111] in cubic notation) and the MnAs precipitates grow perpendicular to the nanowire axis in the [2110] orientation (or [100] in cubic notation).

Variable temperature MFM analysis was conducted on a different nanowire sample at temperatures between 299 K and 344 K. As with the previous nanowire sample, the MFM image of the nanowire at room temperature (Figure 6b) shows dark contrast along the wire. Based on the cross section along the axis of the wire in both the AFM image and MFM image (insets in Figure 6), the dark contrast also corresponds to protrusions from the surface of the wire, an observation that suggests the dark contrast is also due to the presence of the MnAs precipitate. At each temperature, the MFM image was observed to identify any changes in the magnetic contrast (at point a and point b in Figure 5) as the temperature of the sample increases especially above the Tc of the MnAs phase of 318 K. In addition, the cross section along the wire axis of the MFM image was taken to determine the phase shift of the points of interest i.e. points a and b in Figure 6b. The phase shift of these two points were recorded and plotted as a function of temperature (Figure 7). The phase shift of both points showed the same trend; the phase shift decreases with increasing temperature up to 336 K where the phase shift stabilizes to a constant value. This indicates that the MnAs precipitates are still magnetic above the Tc; in addition the nanowire showed increasing dark contrast along its axis at 336 K and 344 K.

IV. DISCUSSION

The growth of (In,Mn)As shell can be considered similar to growing (In,Mn)As on hexagonal InAs because the InAs nanowires have a hexagonal crystal structure with six side facets. Hence, the nanowire shell was grown on different surfaces from that of (In,Mn)As thin films which are typically grown on cubic InAs. Although the growth conditions of the (In,Mn)As thin films were used to grow the (In,Mn)As shell, the growth of (In,Mn)As shell did not yield the same result as the (In,Mn)As thin films; the shell growth led to formation of precipitates on the surface which was identified as MnAs from the TEM analysis. The MnAs precipitates were formed due to the Mn diffusing out of the InAs matrix to form MnAs with the As background.

Figure 6. (a) The AFM image of InAs/(In,Mn)As core/shell nanowire with the inset showing the topographic section analysis along axis of the nanowire, and (b) the MFM image of InAs/(In,Mn)As core/shell nanowire with the inset showing the magnetic section analysis along axis of the nanowire. The dark contrast in the MFM image at the two points labeled "a" and "b" corresponds to protrusions from the surface of the wire. The section analyses show that the two protrusions give negative phase shift in the phase response of the cantilever.
The MFM images showed dark contrast which indicates that the magnetization of the MnAs precipitates is parallel to the magnetization of the tip. In other words, the MnAs precipitates have out-of-plane magnetization i.e. the magnetization is perpendicular to the surface of the sample. The orientation of the magnetic domains was found to be in the [2 1 0] direction, consistent with the easy axis of MnAs. Hence, the magnetic contrast observed in the MFM image is consistent with the easy axis of MnAs.

When MFM was conducted at higher temperatures, the phase shift (magnetic contrast) decreased with increasing temperature. This observation is consistent with the magnetic contrast in the MFM image being due to a magnetic interaction between the tip and the sample; however, there is still some phase shift observed above the $T_c$ of MnAs (318 K) where the ferromagnetic transition should occur, and the MnAs precipitate would no longer be ferromagnetic resulting in zero phase shift (no magnetic contrast). In addition, the phase shift never goes to zero, but from 336 K the phase shift stabilizes at a constant value. This suggests the $T_c$ could be approximately 330 K. At temperatures of 336 K and 344 K, the nanowire showed darker contrast along its axis compared to that of other lower temperatures. The observations of contrast in the entire nanowire is not yet understood, as the $T_c$ of (In,Mn)As is expected to be less than 330 K.

V. CONCLUSIONS

The characterization of the magnetic properties of (In,Mn)As/InAs core/shell nanowires using MFM showed that only the MnAs phase is ferromagnetic. The detection of the MnAs phase using MFM is consistent with the EDS analysis that shows the presence of MnAs precipitates on the surface of InAs nanowire. MFM could not detect the ferromagnetism of the nanowire as a whole; this result is also consistent with the EDS analysis in which Mn was not detected in the InAs
phase. It is possible that the InAs phase contains some Mn, but at a concentration too low to be detected by EDS. In addition, the low concentrations of Mn in InAs phase and small volume of the (In,Mn)As phase could be beyond the limits of detection using our MFM. In the future, the growth conditions of (In,Mn)As shell need to be optimized in order to be able to successfully grow single phase (In,Mn)As shell on InAs nanowires to develop the understanding of nanoscale magnetism in DMS materials.

Acknowledgement
I would like to acknowledge Steve May for the synthesis of the InAs/(In,Mn)As core/shell nanowires, Dr. Jian-Guo Zheng for the TEM analysis, Jon Allen for the preparation of the Au marked SiO2 substrate, Dr. Gajendra Shekhawat for assistance in using AFM, and Ben Myers for assistance in using SEM. The AFM and SEM used within the NUANCE facility were partially supported by Northwestern University’s Murphy Society Undergraduate Research Grant in Nanoscale Engineering. The (In,Mn)As nanowire project is also supported in part by the Materials Research Center and Northwestern University. The use of Materials Research Center Surface Science Facility at Northwestern University is also acknowledged.

REFERENCES
Removing Quantization Artifacts In Color Images Using Bounded Interval Regularization

**Tom Yu Ouyang**
Department of Computer Science

**Jack Tumblin**
Faculty Advisor, Department of Computer Science, Northwestern University

**ABSTRACT**
Coarsely quantized images can exhibit false contours in smooth low gradient regions. Images intended for standard displays such as cathode ray tube (CRT) monitors can show contours when moved to high dynamic range (HDR) devices such as HDR displays and film. While various other methods such as regularization and anisotropic diffusion exist for noise removal and image restoration, they are not able to remove these contouring artifacts completely and can impose substantial blurring. Our method performs iterative regularization within bounded intervals to remove false contours while preserving natural image features.

1. **INTRODUCTION**

   **henever a scene composed of continuous** intensity values is quantized and stored in a digital format, such as when using a digital camera or a scanner, there is inevitable distortion and data loss. Quantization divides the range of input values into a finite number (Q) of non-overlapping "quantization levels" and all input values within a given interval are assigned the same value. For example, most digital images are stored with $Q = 255$ possible values for the intensity at each pixel. When an unquantized image with intensity ranging from 0 to 1.0 is quantized linearly, all values from the original image are mapped to discrete integer values 0-255 in the quantized image.
The noticeable banding in the sky in Figure 1 is a result of coarse color quantization, shown with Q = 16 per color channel. Bands sometimes occur where there are large regions of gradual intensity change in the original unquantized image. When these smooth regions are quantized, the resulting visual effect can be an abrupt and isolated step or "contour" in the displayed image, where pixels on one side of the step are assigned to one quantization level and pixels on the other side are assigned to a neighboring level. We refer to this effect as contouring.

Fortunately, most images available today are stored with enough quantization levels that visible contouring is rare on ordinary CRT or liquid crystal display (LCD). While these images usually do not reveal quantizing artifacts on standard displays, 8-bit contouring can appear on High Dynamic Range (HDR) displays or in cinema and film applications. As higher contrast displays become more common in professional and consumer markets, this problem will become more prominent. This paper proposes a novel method that repairs contouring due to quantization in digital images.

II. MOTIVATION

Repairing quantization errors falls under the broader category of image restoration and reconstruction methods, which attempt to recover an undistorted ideal image \( u(x, y) \) from a distorted observed image \( z(x, y) \), where \( (x, y) \) represent bit coordinates in the image, which we will refer to simply as \( u \) and \( z \). Anisotropic Diffusion Filtering is an iterative method for reducing noise levels while retaining important image information. It relies on measurements of the intensity changes to determine the degree of smoothing to be applied for the given region. Any intensity gradient above a given threshold is considered an edge that exists in the ideal image and is preserved or even enhanced. Given enough iterations, this method generates solutions that may be smooth, but introduces unwanted blurring and loss of image features even when an appropriate edge threshold is chosen, as shown in Figure 2a. Since sharp edges are preserved by the gradient threshold, this over-smoothing occurs in regions of gradual intensity change.

Regularization methods approach image restoration by using prior information about the ideal image (i.e. piecewise or global smoothness) to iteratively improve their estimate of the solution. They also restrict their search to solutions which closely match the observed input \( z \). Tikhonov Regularization\(^2\) generates an estimate for the ideal image \( u \) by minimizing the cost functional \( E \):

\[
E = \int \lambda R(u) + (u - z)^2 \, du
\]  

(1)

The first term \( R(u) \), also know as the regularization functional, measures how well the estimate \( u \) satisfies the prior knowledge about the ideal image. The second term \( (u - z)^2 \) measures the deviation from the observed image. The constant \( \lambda \) controls the balance between the regularization cost and the deviation cost. A common choice for \( R(u) \) is \( |\nabla u|^2 \), which penalizes roughness and favors images that are globally smooth. One improvement to this method is Edge Preserving Regularization\(^1\) which modifies the regularization functional to prevent it from smoothing across sharp edges that should be preserved. Another regularization method, Total Variation\(^3\) uses \( |\nabla u| \) as the regularization functional and usually performs better than the quadratic functional at preserving edges.

A feature of regularization methods is that the cost functional \( E \) penalizes any deviation from the observed data. While this penalty works very well for random noise, it can pose a problem when dealing with contouring due to quantization.
Contouring consists of large bands of the same intensity value separated by abrupt intensity gradients. Since the quantization process discarded all changes inside these bands, the roughness cost is very low and the deviation penalty \((u - z)^2\) dominates the search for the solution. This results in images that are very faithful to the observed input but fail to remove much of the visible contouring, as shown in Figure 2b, even when the regularization process is run for a large number of iterations.

III. PROPOSED METHOD
As mentioned above, Tikhonov regularization penalizes any deviation from the initial observed data when searching for a solution. However, each value stored in a quantized image actually represents a range of possible intensity values that compose the quantization interval. The foundation method proposed by these authors builds upon this idea by treating values in the observed image as a quantization interval instead of a single measurement, as show in Figure 3. Our method only penalizes deviations from the observed input when the resulting intensity exceeds the boundaries of the quantization interval. This greatly relaxes the deviation penalty and permits smoothing only as long as the estimated intensities stay within the quantization level boundaries. Because we know that the ideal unquantized intensity must lie within these boundaries, the method places
a heavy penalty on any deviation that causes the intensity to exceed the quantization interval. This prevents it from over-smoothing both sharp edges and low gradient regions in the image.

In order to incorporate the above modifications into our solution, we use the penalty term \( g(u - z) \). The deviation penalty functional \( g(x) \) is chosen such that it has the following properties.

1. For each pixel in the estimate of the ideal image, there should be no deviation penalty as long as it stays within the original quantization level boundaries, i.e., the pixel belongs to level \( l \), then \( q_l \) and \( q_{l+1} \) are the lower and upper boundaries for that level.

2. If the estimate exceeds \((q_l, q_{l+1})\) boundaries, a strict penalty cost is imposed to force the intensity back within the interval, given by

\[
E = \int \lambda R(u) + g(u) \, du
\]

where

\[
g(u) = \begin{cases} 
0 & \text{if } q_l \leq u \leq q_{l+1} \\
(u - q_{l+1})^2 & \text{if } u > q_{l+1} \\
(u - q_l)^2 & \text{if } u < q_l
\end{cases}
\]

From Figure 3 we can see that contouring in a coarsely quantized image can occur when two neighboring pixel intensities in a smooth region are assigned to different quantization levels. Because it is not possible for a uniformly smooth low-gradient region in the unquantized image \( u \) to become an edge that is larger than one quantization level, we can conclude that deviations \( \gg 1 \) quantization interval were caused by features in the original image \( u \). Based on this approach, our method ignores changes in intensity that exceed one quantization step in order to preserve the edge. However, since our smoothing is already limited by our deviation penalty, the addition of an edge preserving smoothing term provides only some limited visible improvements.

In addition to uniform quantization where each quantization level is the same size, we also need to consider non-uniform quantization where such levels may not be equal. One common source of non-uniform quantization is gamma correction, where more quantization levels are dedicated to the lower (darker) intensities and fewer to the higher (brighter) ones. Our method handles non-uniform quantization by varying values for \( q_l \) to match the quantization scheme.

IV. IMPLEMENTATION

To minimize the cost functional \( E \) from Equation 2, our method uses an iterative gradient descent technique. We use the discrete membrane model in Equation 3 proposed by Blake and Zisserman combined with the deviation penalty functional \( g(u) \) defined in the previous section. In order to improve the convergence rate our implementation performs Successive Over Relaxation, overcorrecting by a factor \( \omega \) every iteration. Based on our experiments, the method was able to converge within 20 iterations for a 512×512 color image.

\[
E = \lambda \{(u_{k+1} - u_k)^2 + (u_{k+1} - u_k)^2\} + g(u_k)
\]

For each pixel intensity in the current iteration \( u_{k+1} \), we use the gradient of the cost \( E \) in Equation 3 to determine the intensity in the next iteration \( u_{k+1}^{(k+1)} \).

\[
u_{k+1}^{(k+1)} = u_{k+1}^{(k)} - \omega \frac{dE}{du_{k+1}^{(k)}}
\]

Combining Equations 3 and 4, we arrive at the following implementation for the gradient descent algorithm.

For each pixel \( u_{k+1}^{(k)} \), if \( q_l \leq u_{k+1}^{(k)} \leq q_{l+1} \),

\[
u_{k+1}^{(k+1)} = \begin{cases} 
u_{k+1}^{(k)} & \text{if } u_{k+1}^{(k)} < q_l \\
\nu_{k+1}^{(k)} - \omega \{4u_{k+1}^{(k)} - (u_{k+1}^{(k)} + u_{k+1}^{(k)} + u_{k+1}^{(k)} + u_{k+1}^{(k)})\} \end{cases}
\]

Otherwise, if \( u_{k+1}^{(k)} < q_l \), \( q_l = q_{k+1} \), and if \( u_{k+1}^{(k)} > q_{l+1} \), \( q_{l+1} = q_{k+1} \),

\[
u_{k+1}^{(k+1)} = \begin{cases} \nu_{k+1}^{(k)} - \omega \{1 + 4\lambda\}u_{k+1}^{(k)} - q_l & \text{if } u_{k+1}^{(k)} > q_{l+1} \\
\nu_{k+1}^{(k)} - \omega \{1 + 4\lambda\}u_{k+1}^{(k)} + u_{k+1}^{(k)} + u_{k+1}^{(k)} & \text{if } u_{k+1}^{(k)} < q_l
\end{cases}
\]

V. RESULTS

The simulation results from the proposed method show a significant improvement in both contour removal and edge preservation in the restored images. Figure 4 provides a scan-line comparison between our method (top), edge preserving
regularization (middle), and anisotropic filtering (bottom). The figure shows that our method is able to generate a smooth estimate of the ideal image which stays completely inside the quantization boundaries while edge preserving regularization was not able to completely remove the contours in the image and still shows partially smoothed steps. In both edge preserving regularization and anisotropic diffusion there are areas where the solution deviates from the quantization boundaries which is inconsistent with the ideal image. While the result of anisotropic diffusion provided a smoother curve with less significant contours, there are significant deviations from the original input. Since the gradient on the right side of the scan-lines is not sharp enough to trigger the edge threshold, both methods over-smoothed that region. Figure 5 shows the result of our method on two coarsely quantized images with very visible contouring artifacts. The reconstruction we produced removes the contours in the sky while maintaining detail in the rest of the image.

Figure 4. The scan-line graph from solution generated by (top) the proposed method, (middle) edge-preserving regularization, and (bottom) anisotropic diffusion. The quantization interval boundaries are also included for reference.
VI. DISCUSSION

This paper develops a novel regularization approach to repair contouring artifacts on color quantized images. Our approach treats the intensities of the input image as an interval instead of individual measurements. This approach allows removal of contouring artifacts that are preserved by other methods.1,2

The proposed method is most effective for images where the only significant error is due to quantization and is not intended as a general noise removal algorithm. For noisy images, it is possible to relax the deviation penalty $g(x)$ from Equation 3 to better handle to noise. It may also be possible to apply another algorithm such as Total Variation regularization$^3$ to first remove the noise before using our method to repair any contouring artifacts.

REFERENCES


---

Figure 5. Reconstruction of coarsely quantized images.
Coarsely quantized image with (left) $Q=16$ restored to (right) $Q=255$ using the proposed method.
Evaluating Sam: Iterative Design of a Story Listening System and Embodied Conversational Agent

Candice W. Tse
DEPARTMENT OF COMPUTER SCIENCE,
DEPARTMENT OF COMMUNICATION STUDIES

Justine Cassell
FACULTY MENTOR, DEPARTMENT OF COMPUTER SCIENCE,
DEPARTMENT OF COMMUNICATION STUDIES, NORTHWESTERN UNIVERSITY

ABSTRACT

Sam is an Embodied Conversation Agent and Story Listening System designed to improve children's literacy by engaging them in storytelling and collaborative play. This research aims to develop techniques to evaluate the new technology of an ECA combined with a SLS. The results of this study found that evaluating Sam requires an understanding of children's learning theory, elements of ECA design, and the coding of children's narrative discourse. Each of these three techniques are explained in detail and applied to the Sam project. This paper discusses the concepts of collaboration and scaffolding in children's narrative, describes the ideal of beauty as a crucial measure of ECA design success, and reviews the coding scheme devised to assess our results. Furthermore, as part of a continual process of iterative design and evaluation, future implementation ideas for Sam are presented.

1. INTRODUCTION

With stories about vampires, princesses, puppies and squirrels to keep us company, evaluating Sam, an Embodied Conversational Agent (ECA) and Story Listening Systems (SLS), is never dull. Essentially, Sam represents an effort to allow children to create their own meaningful content and improve literacy through collaborative play. Accomplishing this task requires an understanding of SLS concepts such as emergent literacy, and scaffolding and children's behaviors, and uses an interdisciplinary approach incorporating Computer Science, Psychology, Cognitive Science and Communication Studies. Sam is a combination of two technologies: Embodied Conversational Agents (ECAs) and Story Listening Systems (SLS). ECAs are visual com-
puter interfaces that act as social beings through the ability to employ behaviors such as emotion, personality, performatives and conversational function.¹

Story Listening Systems (SLS) are implementations of a Model of Technology and Literacy Development incorporating four essential traits: (1) use storytelling to bootstrap literacy; (2) encourage peer play; (3) engage children in normal play behaviors instead of requiring them to learn computer desktop skills; and (4) ask children to construct their own personally meaningful content. These systems allow children to succeed at three crucial predictors of literacy: (1) using decontextualized language; (2) collaborating with peers to make meaning; and (3) gaining metalinguistic awareness.²

Sam is a virtual child who invites children to participate in collaborative and conversational storytelling play with real toys. Sam is projected on a plasma screen behind a wooden toy castle, and can both listen to a child’s stories and tell her own. (Figure 1) Figurines that can exist in either the physical world or on the screen allow Sam and the child to pass the story back and forth between their worlds.

Although methods exist to evaluate ECAs and SLS separately, a new set of techniques is required to accurately appraise Sam, which is both an ECA and a SLS. Not only is an understanding of children’s learning theory required, but elements of ECA design and coding for children’s narrative discourse need to be factored in to fully assess both the educational effectiveness and usability of Sam.

The discussion of these concepts will be done in the following order. First, children’s learning theory will be explored, with a focus on collaboration and scaffolding. Second, we will analyze the concept of beauty as it pertains to ECA design—specifically, the qualities of accuracy and user satisfaction. Third, the coding of children’s narrative discourse and a hypothesis about our results will be examined. Finally, the culmination of this analysis will result in suggestions for the future improvement of Sam.

II. METHODS

Evaluating Sam has three components: 1) theoretical research on children’s literacy and collaboration, 2) iterative design of an ECA, and 3) coding of children’s narrative discourse. Integrating these three processes offers a well-rounded view of the project as well as the opportunity to apply the knowledge gained in one component to support and improve another component. Iterative design is the concept that a system should be designed cyclically; that is, a constant cycle of design, testing, and re-modification is expected and is considered part of the design process. This philosophy is essential to the Sam project because of its exploratory nature; there are always improvements that can be made.

Learning Theory

Certain features of children’s learning theory are essential to understanding the motivation behind Sam. These features include collaboration and scaffolding.

Collaboration between peers has been shown to improve the complexity of children’s
stories when compared to children performing solo narrations. Bokus' research found that for five and six year olds, texts that were co-narrated resulted in consistently higher scores, and that complex episodes occurred more frequently in the co-narration case than during solo narration.

Scaffolding refers to the act of providing a framework for children to narrate within, while still allowing for creative story creation. Researchers have found that the use of scaffolds, such as scripts or leading questions, by more capable peers or teachers have resulted in more complex collaborative stories. Furthermore, collaboration between peers of different literacy levels can have positive influences on the learning ability of the less capable peer.

Sam builds upon these key concepts in order to improve children's literacy. In experimental situations, Sam and the child can each tell solo-narrated stories or collaborative ones. In the latter case, Sam starts telling the story and elicits a continuation from the child through natural turn-taking behaviors. As the more competent peer, Sam presents a scaffolding framework for the child to provoke more complex storytelling. Finally, interaction with Sam and the wooden castle artifact allow literacy skills to emerge through improvisational play.

**ECA Design**

Nass et al. describe three criteria for “beauty” in ECA design: 1) accuracy; 2) user satisfaction; and 3) peer admiration. These criteria determine the success of an ECA design and represent the pinnacle of this emerging field. Accuracy is described as “the extent that the agent’s outward appearance objectively matches the appearance, language, attitudes and behavior of humans,” and is determined in our case by Sam’s appearance, features, gestures, and stories. Second, an ECA must satisfy its diverse users. Additionally, peer admiration is a qualitative measure of the opinions of other scholars in the community; they generally agree on what is “interesting” in their field.

**Accuracy**

Further investigation into the design of ECAs inevitably requires a solid understanding of the wide spectrum of human behaviors that occur during conversation, such as gestures, speech, facial expressions, and eye gaze. Integrating gestures and speech at the appropriate temporal junctures is important in order to enhance the effects and the believability of the ECA. This is particularly relevant in the construction of Sam’s stories, because precise and exact movements are necessary to make Sam seem natural to the user. We found through trial and error that even slight missteps in timing resulted in unnatural and awkward movements that distracted the user from the story itself.

During the story scripting process, we determined that we needed to implement more features, such as facial expressions, gestures, and utterances, in order to enhance Sam’s believability. These developments will enhance the accuracy of our ECA, because varied facial displays and movements are important conversational tools.

One current and serious limitation of all ECAs involves natural language processing and the ability of an ECA to provide believable and realistic responses. This is particularly problematic because children’s speech is inadequately supported by current speech recognition technology. Because of this deficiency, Sam relies on pre-recorded speech and pre-scripted actions. Shortcomings of this approach include an inability to respond to certain speech acts, such as a child’s questions or actions. In addition, responses eventually become repetitive due to their limited nature. However, this limitation exists on a much greater scale than this project alone, and is related to two fields.

For many years, the areas of Machine Translation (MT) and Natural Language Processing (NLP) have been working towards solutions to the problems computers have with understanding languages. Yet, researchers estimate at least another twenty years before fully comprehensive MT will be available to the world. As it stands, translation engines aim to have three characteristics: to be general purpose, high quality, and fully automatic. Current research has yielded engines that can embody only two of the three characteristics, because a trade-off exists between shallow translation (fast, surface-level dictionary lookup which can be applied to a wide range
of options) and deep translation (much more accurate, but limited in domain).

For example, the popularity of Multi-User Dungeons (MUDs) in the 1990's, text-based, online fantasy spaces where users could participate in role-playing games and chat with other users, spurred the creation of autonomous agents named MUD bots. These bots had the ability to respond to user’s questions over the text-based medium and pose as real human beings in these online chats. Using simple pattern-matching rules, one such bot, Julia, was able to fool numerous users in lengthy conversations. Eventually, the users found out that Julia wasn’t real, therefore the bot failed the Turing test. However, the promise that such a simple autonomous agent provides for ECAs such as Sam should be encouraging. Since Sam exists in the limited domain of children’s discourse, it appears possible that as the technological barriers of voice recognition and speech processing are lifted, more flexibility will become available in terms of Sam’s reliance on pre-recorded audio.

**User Satisfaction**

Nass et al. discuss the concept of “varied conditions,” which concerns an ECA’s ability to respond to any experimental situation. With the unpredictability of children in particular, this idea is especially relevant to Sam. Included in these conditions are uncontrollable elements such as a subject’s personality, literacy level, and background, all of which need to be equally catered to by a well-designed ECA. Without taking the wide range of possible subjects into account, an ECA would fail to satisfy this essential criterion to beauty.

Nass et al. also focus on ethnicity and race as it pertains to an individual’s opinion formation process. They found that participants related more to agents of their own ethnicity and race compared to those of different races, and rated them higher on scores of similarity, social attractiveness and trustworthiness. To avoid this effect, Sam is designed to be race and gender neutral.

Studying SLS and ECAs also relates to the degree to which the children felt that Sam was “real.” Turkle’s study on childhood animism, in which children playing with a mechanical tic-tac-toe toy, Merlin, believed that the toy had feelings, raises questions about the effects an ECA like Sam could have on children. Some children involved in the study insisted that Merlin was alive because it would “cheat” and win, while others claimed that it wasn’t alive because it didn’t have a mother or eyes. The children would, however, develop social relationships with their toys by anthropomorphizing them and giving them human emotions.

As this study related only to mechanical toys that were clearly not human, one must consider whether this occurs when children play with Sam, who resembles a human form and sounds just like a child. As the criteria for a “beautiful” ECA includes the necessity that it “satisfy the users of the technology,” the degree to which children believe that Sam is “real” may have an effect on the level of satisfaction they experience.

**Coding Children’s Narrative Discourse**

Our hypothesis about Sam is that the effects that collaboration and scaffolding have on the resulting measures of emergent literacy will result in increased complexity of the children’s stories. The success of SLS is reflected in the stories told by children who interact with Sam. The final component of evaluating Sam, therefore, involves applying a coding scheme to the stories. Our coding scheme’s purpose is to quantify the components of children’s narrative discourse and allow researchers to accurately compare the results across different stories.

It is easy to overlook the inherent complexity of simple children’s narratives, so a clear and realistic coding scheme design is essential for evaluating the stories. The analysis of the stories provides researchers with quantitative data that is easily comparable across stories, and creates a standardized method of comparison for future analyses. We devised a narrative structure coding scheme based on similar ones developed by Bokus, Trabasso and Stein/Policastro. The basic unit in the scheme is the reference situation, a single narrative element containing a subject and a state. Building from this basic unit, our scheme awards an increasing number of points for temporal sequencing, cause and effect, a goal, actions taken
to reach the goal, and the arrival at an outcome. The highest score is awarded to a story that has two complete episodes (attempt and outcome) which share a common goal. This process is essential to determining the effectiveness of Sam, and provides a more well-rounded view of the whole project.

To design and test our coding scheme, we used stories collected from children interacting with the Sam system. Participants were thirty-one kindergarteners from two classes at an elementary school in the Boston area. Their ages ranged from five years, one month to six years, ten months. Nineteen girls and twelve boys participated in the IRB-approved study. We believe this coding scheme will enable us to quantitatively compare the narrative structure of children's stories in future studies on Sam.

III. RESULTS
As the purpose of this research is exploratory and designed to produce a structure for evaluating ECA and SLS systems, the results of this study produced three techniques for evaluation: 1) theoretical research on children's literacy and collaboration, 2) iterative design of an ECA, and 3) coding of children's narrative discourse.

Because ECA design is a decidedly nascent field, improvements are absolutely essential. By applying these evaluation techniques to the Sam system, we determined what changes need to be made. Improvements to Sam's Wizard of Oz (WoZ) interface, such as incorporating a touch screen, creating a more intuitive and consistent visual metaphor for the user interface, and allowing for keyboard shortcuts, would enable the researcher to control Sam more efficiently. Furthermore, designing more actions and expressions could allow Sam to present a more convincing and engaging image to the user. Furthermore, developments in the fields of Artificial Intelligence, namely Natural Language Processing and Machine Translation, could allow ECAs to extend their reach to different cultures in a more convincing and realistic way, and remove the need for a WoZ altogether.

One improvement to the coding process would allow the coder to see the actual videos of the children interacting with Sam in parallel with the transcription. Doing so would provide the coder with the further context of analyzing social behavior in the stories, and present a richer environment in which to work.

IV. DISCUSSION
Since its inception purely as a solo narration story tool, Sam has taken on numerous other purposes and forms, one of which is a collaborative version of Sam. This version tells stories and models the collaborative roles children use when storytelling together, to encourage collaboration between Sam and the child within a structured story environment. Sam builds stories collaboratively with the child rather than telling a complete story. By developing the system to better model children's storytelling behaviors, we hope to make Sam more engaging and increase its value as an educational tool.

Although much more work needs to be done on Sam, the implications of such research have far reaching educational potential. Continued research in the field of ECA and SLS is required to develop new techniques for evaluating the effectiveness and usability of such systems. Since this is a new and uncharted field of study, unique methods of evaluation are essential to furthering the development of these technologies. With full knowledge that our design process is of an iterative nature and in itself a learning process, we strive toward the ultimate goal for ECA design: beauty.

Acknowledgements
Many thanks go to Prof. Justine Cassell and Andrea Tartaro for making this paper possible. They encouraged, pushed, and helped me every step of the way. Also, I'd like to thank all the members of the Articulab for all their hard work, and Kim Ferriman for providing the chocolate.
REFERENCES


ABOUT THE CONTRIBUTORS

AYSHA CHOWDHRY
History and Asian/Middle Eastern Studies, 2004

Aysha Chowdhry graduated in 2004 with a double major in History and Asian/Middle Eastern Studies from the Weinberg College of Arts and Sciences. She completed her research project in the Physics Department with Dr. Arthur Schmidt and fellow author Elaine Tsao. She is currently completing a master's degree in Near and Middle Eastern Studies in London. Aysha hopes to return to the states soon and enter a doctorate program in history.

MICHAEL CARNEY
Electrical Engineering, 2006

Michael Carney is a senior Electrical Engineering student in the McCormick School of Engineering. He began his work in June 2004 at NASA Kennedy Space Center (KSC) under the direction of Dr. James C. Simpson and completed it in late September 2004. Since completion, Michael worked on Virgin Atlantic’s GlobalFlyer project, the first solo around-the-world flight on a single tank of gas. Michael will be continuing work at KSC this summer and fall as a co-op student and hopes to acquire a full-time position there after graduation in June 2006.

KWAN Y. CHEN
Biomedical Engineering, 2005

Kwan Chen is a senior Biomedical Engineering student in the McCormick School of Engineering. Blending engineering with medicine has been Kwan Y. Chen’s goal since coming to Northwestern University. After sophomore year, he began working with Dr. Anjali Chenn at the Feinberg School of Medicine, where he was able to acquire further knowledge in medicine through cancer research. This opportunity has allowed Chen to learn molecular biology techniques and apply them to quantifying the motility of fibroblast tumor cells. When he graduates, he plans to continue extensive research in medicine.

DINNA GERALDINE RAMLAN
Materials Science & Engineering, 2005

Dinna Ramlan is a senior Materials Science and Engineering student in the McCormick School of Engineering. This published work was done in the Department of Materials Science and Engineering under the supervision of Professor Lincoln Lauhon. Dinna is still doing research in the characterization of InAs/MnAs nanostructures with Professor Lauhon as a continuation of the published work.

JAMES M. RONDINELLI
Materials Science & Engineering, 2006

James M. Rondinelli is a junior Materials Science and Engineering student in the McCormick School of Engineering. The research James presented here was completed while working as an associate member of the Photonic Theory Group at the University College Cork Tyndall Institute during the summer of 2004. He is currently researching the computational modeling of surface structures with Prof. Laurence Marks of the High Resolution Electron Microscopy and Surface Structure Facility at Northwestern University.

BENJAMIN JAY STEPHENS
Mechanical Engineering, 2006

Benjamin Jay Stephens is a junior Mechanical Engineering student in the McCormick School of Engineering. His research is being performed in the Laboratory for Intelligent Mechanical Systems (LIMS) at Northwestern University under the supervision of Kevin Lynch. He is currently working on a research project funded by a Ford Undergraduate Research Grant to develop a robot capable of adaptive locomotion in a tunnel-like environment for use in search and rescue or exploration applications.

CANDICE W. TSE
Major: Economics & Communications Studies, 2006
Minor: Computing Information Systems

Candice Tse is a junior studying Economics in the Weinberg College of Arts and Sciences and Communication Studies in the School of Communication. Her paper is based on independent research conducted under Professor Justine Cassell in both the fields of Communication Studies and Computer Science in the McCormick School of Engineering. She is currently continuing research into the Sam project with a focus on improving the user interface through the study of Human-Computer Interaction.

TOM YU OUYANG
Computer Science, 2005

Tom Ouyang is a senior Computer Science student in the McCormick School of Engineering. This research was done in the Computer Science Department under the direction of Professor Jack Tumlin. He is currently working in the Qualitative Reasoning Group on the use of analogy in problem solving.